



THEORETICAL ANALYSIS OF GEOMAGNETIC MICROPULSATIONS AND ELECTROMAGNETIC INDUCTION IN A LAYERED EARTH

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ABSTRACT

Geomagnetic micropulsations are small, quasi-periodic variations of the Earth's magnetic field that are closely coupled to electric currents in the ionosphere, magnetosphere and conducting Earth. This paper develops a compact theoretical framework for the electromagnetic response of a layered Earth to micropulsation-frequency fields. The analysis starts from Maxwell's equations and uses a separation of variables representation for a horizontally structured electromagnetic field in a conducting half-space. Neglect of displacement current, justified at micropulsation frequencies for a sufficiently conducting Earth, reduces the vertical field equation to a diffusion-type electromagnetic equation. The resulting skin-depth relation provides a direct measure of the penetration depth and explains why long-period pulsations are required for deeper electrical sounding. A two-layer Earth model is then considered to relate surface electric and magnetic fields to the conductivity and thickness of an upper layer over a basement. The role of the horizontal separation constant is also discussed as a measure of source-field scale. Finally, the effect of a sharp conductivity discontinuity, represented by a coastline, is examined. Such a discontinuity can enhance the vertical magnetic component because induced currents must redistribute near the boundary. The framework provides a physically transparent basis for vertical conductivity profiling and for interpreting anomalous micropulsation fields in regions with strong lateral conductivity contrasts.

KEYWORDS: geomagnetic micropulsations; magnetotelluric field; electromagnetic induction; layered Earth; skin depth; conductivity profile; coastline effect.

1. Introduction

Geomagnetic micropulsations are short-period magnetic variations produced by time-dependent electrodynamic processes in near-Earth space. When these external magnetic variations reach the Earth, they induce electric fields and telluric currents in the conducting crust and upper mantle. The coupled electric and magnetic fields are commonly treated as a magnetotelluric field. An important consequence is that surface measurements of horizontal electric and magnetic components contain information about the electrical conductivity distribution beneath the observing station. The theoretical basis of this relation was developed in classical magnetotelluric treatments by considering a conducting half-space and subsequently replacing the continuous conductivity distribution by practical layer models. [4,7,8,27]

The present work focuses on two aspects that are especially useful for micropulsation studies: (i) vertical profiling of electrical conductivity using the surface relation between tangential electric and magnetic fields, and (ii) modification of the magnetic response by horizontal conductivity discontinuities. The treatment follows the separation-of-variables approach described in the supplied source material, while reorganizing the derivation into a concise research-paper format. The objective is not to introduce new observational data, but to establish the equations and physical interpretation needed to connect micropulsation measurements with a layered-Earth response. [1,7]

For a Cartesian coordinate system, the Earth's surface is taken as $z = 0$ and the conducting Earth occupies $z > 0$, with z increasing downward. The conductivity is initially assumed to vary only with depth, $\sigma = \sigma(z)$. This one-dimensional assumption makes it possible to isolate vertical electromagnetic diffusion and to demonstrate how surface fields depend on subsurface conductivity.

2. Theoretical Formulation

2.1 ELECTROMAGNETIC FORMULATION

The electromagnetic field inside the Earth is governed by Maxwell's equations. With $D = \epsilon E$, $B = \mu_0 H$ and $J = \sigma(z)E$, the relevant equations are

$$\nabla \cdot D = 0 \quad (1)$$

$$\nabla \cdot B = 0 \quad (2)$$

$$\nabla \times E = -\partial B / \partial t \quad (3)$$

$$\nabla \times H = J + \partial D / \partial t \quad (4)$$

Assuming harmonic time dependence of the form $\exp(i\omega t)$, Faraday's law becomes

$$\nabla \times E = -i\omega\mu_0 H \quad (5)$$

Combining Faraday's law with Ampère's law gives the wave equation for the electric field. For a homogeneous magnetic permeability, it can be written as [1,18]

$$\nabla^2 E = i\omega\mu_0\sigma E - \omega^2\mu_0\epsilon E \quad (6)$$

Equation (6) contains conduction and displacement-current contributions. At micropulsation frequencies the displacement-current term is normally much smaller than the conduction-current term in the conducting Earth. The low-frequency approximation therefore gives

$$\nabla^2 E \approx i\omega\mu_0\sigma E \quad (7)$$

This approximation is central to magnetotelluric induction at long periods because the response is controlled primarily by conduction current rather than by electromagnetic-wave propagation through free space.

2.2 SEPARATION OF VARIABLES AND FIELD STRUCTURE

The current-continuity condition follows from the divergence of Ampère's law together with charge conservation. For $\sigma = \sigma(z)$, the field can be represented in a form that separates horizontal source structure from vertical induction. Let

$$\mathbf{E} = F(x,y) Z(z) \exp(i\omega t) \quad (8)$$

A convenient divergence-free horizontal vector is obtained from a scalar function $P(x,y)$:

$$\mathbf{F}(x,y) = (\partial P/\partial y, -\partial P/\partial x, 0) \quad (9)$$

The horizontal function then satisfies a Helmholtz-type equation

$$\partial^2 P/\partial x^2 + \partial^2 P/\partial y^2 = -\alpha^2 P \quad (10)$$

where α is the horizontal separation constant. The corresponding vertical equation is

$$d^2 Z/dz^2 = [\alpha^2 + i\omega\mu_0\sigma(z)] Z \quad (11)$$

when displacement current is neglected. The parameter α describes the horizontal scale of the inducing source: small α corresponds to a large horizontal source scale, while larger α represents a more localized current system. Thus α is not simply a mathematical constant; it provides a useful measure of source geometry in the interpretation of surface induction.

For the separated solution, the electric field has only horizontal components, while the magnetic field contains components generated by the spatial variation of the electric field. In particular, the vertical magnetic component is proportional to the horizontal structure represented by α . Hence a laterally uniform plane-wave limit, $\alpha \rightarrow 0$, has a substantially different vertical-field behavior from a finite-scale source.

2.3 LOW-FREQUENCY INDUCTION AND SKIN DEPTH

For a uniform conducting layer, the vertical equation can be written in terms of the complex propagation parameter θ . In the plane-wave limit $\alpha = 0$,

$$\theta^2 = i\omega\mu_0\sigma \quad (12)$$

Choosing the physically decaying solution gives $\theta = (1+i)/\delta$, where the skin depth is

$$\delta = \sqrt{2/(\omega\mu_0\sigma)} \quad (13)$$

The skin depth δ is the characteristic distance over which the amplitude of an electromagnetic disturbance decreases substantially inside a conductor. Equation (13) shows two important trends: increasing frequency decreases penetration depth, while increasing conductivity also decreases penetration depth. Conversely, long-period micropulsations penetrate more deeply and are therefore more useful for probing deeper electrical structure. [1]

The exponential depth dependence can be represented schematically as

$$Z(z) \propto \exp(-z/\delta) \exp(i z/\delta) \quad (14)$$

The first factor describes attenuation and the second represents phase change with depth. For the representative conductivity scale quoted in the source material, $\sigma \approx 10^{-1} \text{ S m}^{-1}$, the characteristic skin depth is of order tens of kilometres at sufficiently low angular frequency and becomes much smaller as frequency increases. The exact depth reached by a micropulsation depends on both period and conductivity; therefore a range of periods is preferable when constructing a conductivity profile.

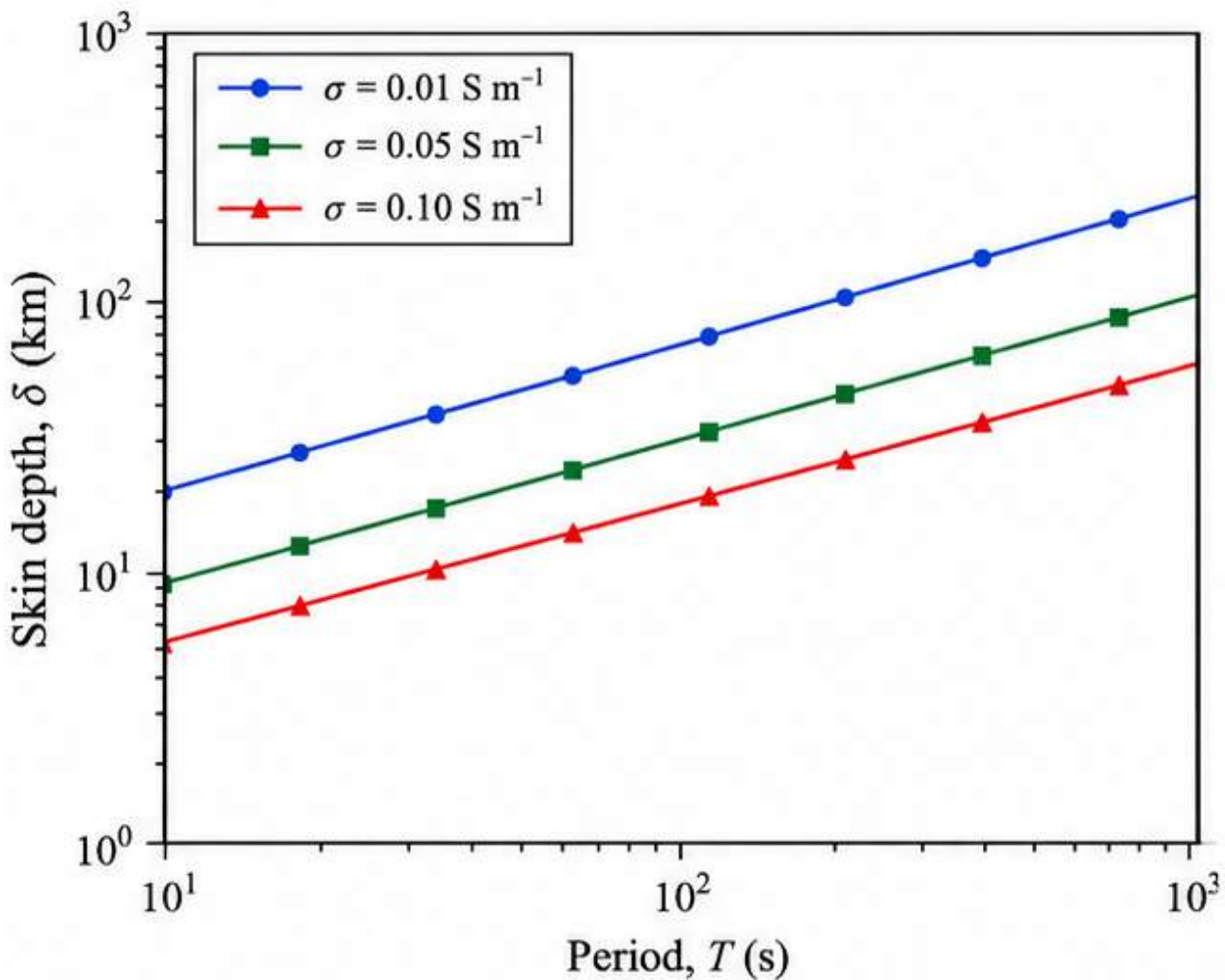


Figure 1: Variation of Skin depth with micropulsation period.

2.4 SURFACE IMPEDANCE AND VERTICAL CONDUCTIVITY PROFILING

The principal observable in magnetotelluric sounding is the ratio between tangential electric and magnetic fields at the Earth's surface. At $z = 0$, continuity of tangential E and B across the air-Earth interface provides the boundary conditions required to connect the external inducing field to the internal conducting response. In the $\alpha = 0$ plane-wave limit, the surface relation takes the familiar impedance form [1,7]

$$Z_e(\omega) = E_x(\omega) / H_y(\omega) \quad (15)$$

or, when the magnetic quantity is expressed as $B_y = \mu_0 H_y$,

$$E_x(\omega) / B_y(\omega) = Z_e(\omega) / \mu_0 \quad (16)$$

For the simple homogeneous half-space, the magnitude of the impedance is proportional to $\sqrt{(\omega \mu_0 / \sigma)}$. Thus, measurements at different frequencies can be interpreted in terms of apparent conductivity or resistivity. In a layered Earth, however, the surface impedance is controlled by reflections and transmissions between layers and therefore carries information about layer thickness as well as conductivity.

Consider an upper layer of conductivity σ_1 and thickness D over a basement with conductivity σ_2 . For each layer, define

$$\theta_j = \sqrt{(\alpha^2 + i\omega\mu_0\sigma_j)}, \quad j = 1, 2 \quad (17)$$

The vertical solution in a finite upper layer is a combination of upward- and downward-decaying

terms. Boundary matching at $z = 0$ and $z = D$ gives the surface impedance. For the plane-wave case $\alpha = 0$, the upper-layer parameter reduces to $\theta_1 = (1+i)/\delta_1$. A compact recursive form for the impedance of a two-layer system is

$$Z_1 = Z_0 [Z_2 + Z_0 \tanh(\theta_1 D)] / [Z_0 + Z_2 \tanh(\theta_1 D)] \quad (18)$$

where Z_0 denotes the intrinsic impedance of the upper layer and Z_2 the impedance looking into the basement. Equation (18) is useful because it shows explicitly how the measured surface response depends on upper-layer thickness D , conductivity through θ_1 , frequency through ω , and the basement impedance.

The practical profiling procedure follows directly from this dependence. First, a micropulsation with a sufficiently well-defined frequency is selected and the surface electric and magnetic amplitudes are measured. Second, a trial conductivity is assigned to the upper layer. Third, the theoretical surface response is calculated for a range of D and α . Finally, the measured impedance is compared with the model response and the combinations of thickness, conductivity and source scale that reproduce the observations are identified. Repeating the procedure over several frequencies reduces ambiguity and improves the physical interpretation of the depth profile. [1,14]

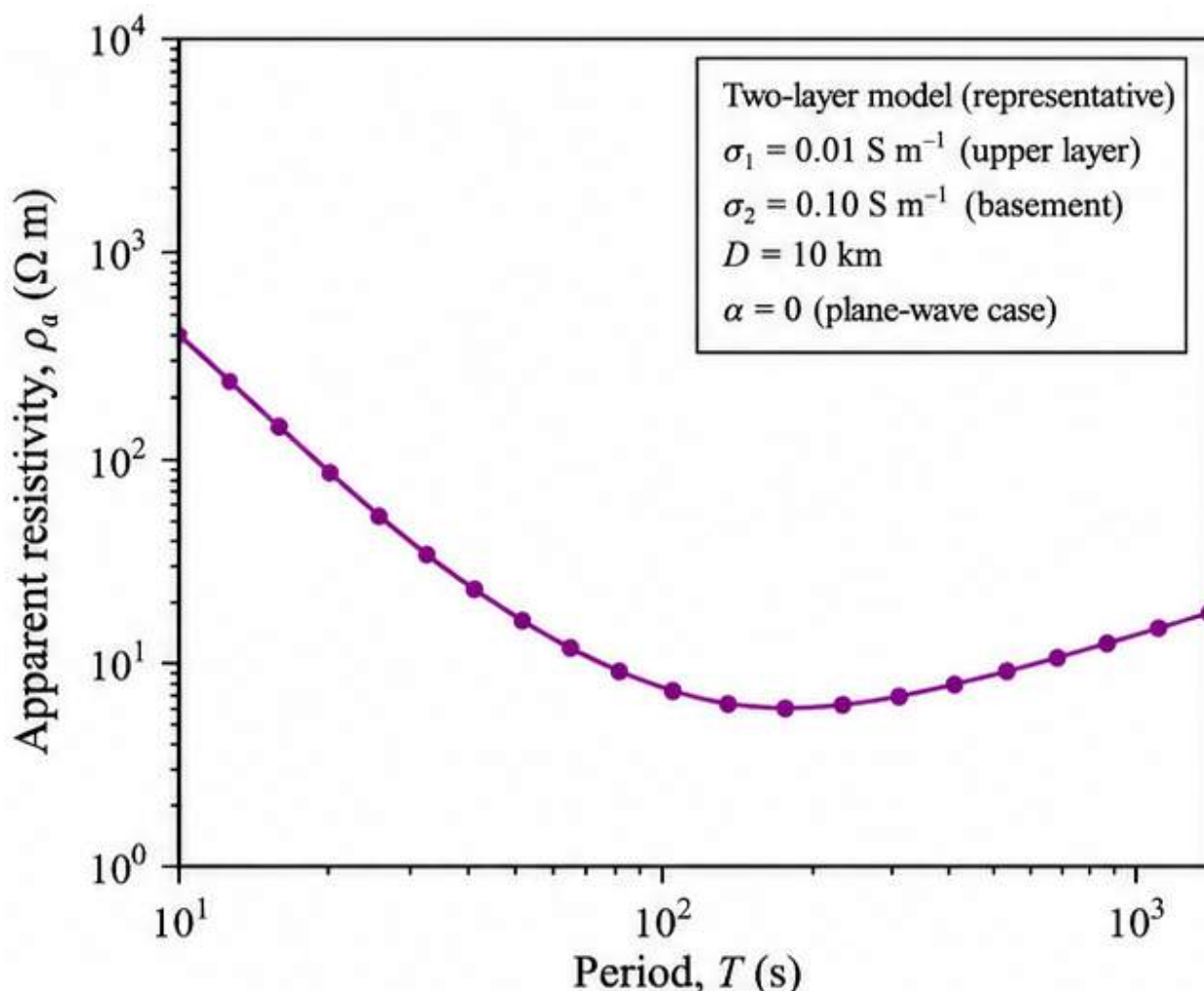


Figure 2: Apparent resistivity variation with micropulsation period.

Table1:Skin depth values with micropulsation period for different conductivities

Skin depth for

Period T (s)	Frequency (Hz)	$\sigma = 0.01 \text{ S m}^{-1}$ (km)	$\sigma = 0.05 \text{ S m}^{-1}$ (km)	$\sigma = 0.10 \text{ S m}^{-1}$ (km)
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10	0.100	15.9	7.1	5.0
30	0.0333	27.6	12.3	8.7
60	0.0167	38.9	17.4	12.3
100	0.0100	50.3	22.5	15.9
300	0.00333	87.1	39.0	27.6
600	0.00167	123.2	55.1	38.9
1000	0.00100	159.2	71.2	50.3

Table 2. Theoretical apparent resistivity versus micropulsation period

Sl. No. Period T (s) Frequency f (Hz) Apparent resistivity $\rho(\Omega \text{ m})$ **1 10 0.100000 83.58****2 20 0.050000 59.03****3 30 0.033333 47.72****4 50 0.020000 36.91****5 70 0.014286 31.55****6 100 0.010000 27.07****7 150 0.006667 23.16****8 200 0.005000 20.98****9 300 0.003333 18.54****10 500 0.002000 16.28****11 700 0.001429 15.16****12 1000 0.001000 14.20**

2.5 TWO-LAYER EARTH: PHYSICAL INTERPRETATION

The two-layer model is the simplest practical representation of a non-uniform Earth. The upper layer may represent a relatively resistive crustal section, while the basement represents a more conductive or more resistive deeper region. Although the model is idealized, it captures the principal role of conductivity contrasts in controlling induced current flow. [1,7]

The dimensionless quantity $\theta_1 D$ is particularly informative. When $|\theta_1 D|$ is small, the electromagnetic field penetrates through most of the upper layer and the measured response is strongly influenced by the basement. When $|\theta_1 D|$ is large, the field is attenuated substantially within the upper layer and the basement contribution becomes weaker. Consequently, a frequency sweep effectively changes the depth sensitivity of the measurement.

$|\theta_1 D| \ll 1 \rightarrow$ strong sensitivity to deeper structure (19)

$|\theta_1 D| \gg 1 \rightarrow$ response dominated by the upper layer (20)

This interpretation explains why a single period is generally insufficient to determine a unique conductivity-depth model. Long-period fields sample greater depths, whereas shorter-period fields emphasize shallower material. A layered inversion should therefore use multiple frequencies and, where possible, both electric and magnetic components.

2.7 EFFECT OF HORIZONTAL CONDUCTIVITY DISCONTINUITIES

The one-dimensional model assumes that conductivity is horizontally uniform. Natural Earth structure frequently violates this assumption. A particularly strong example is a coastline, where seawater is highly conductive compared with many crustal rocks. The conductivity contrast forces induced currents to redistribute laterally near the boundary.

For a laterally uniform Earth, the fluctuating vertical magnetic component is generally small in the idealized plane-wave limit. Near a conductivity discontinuity, however, current continuity and electromagnetic boundary conditions require the induced current system to bend and concentrate. The associated secondary magnetic field can therefore acquire a significant vertical component.

The effect is strongest when the electric vector of the inducing field is parallel to the discontinuity. In the idealized coastline model described in the source material, the enhancement is particularly sharp on the seaward side and becomes more spatially extended on the landward side. The precise amplitude and spatial scale depend on frequency, conductivity contrast, source geometry and coastline orientation. [5,9,19]

$\nabla \cdot \mathbf{J} = 0 \rightarrow$ lateral redistribution of induced current near $\sigma_1 \neq \sigma_2$ (21)

Because the vertical magnetic component can be generated by lateral current redistribution, an observed increase in B_z should not automatically be interpreted as evidence for a vertically varying conductivity profile. Near coasts and other strong geological boundaries, a two-dimensional or three-dimensional treatment may be required. This distinction is important in practical geomagnetic micropulsation analysis.

3. Discussion

The theoretical results establish a direct chain between external micropulsation fields and subsurface electrical structure. Maxwell's equations determine the induction process; the separation constant α represents the horizontal scale of the source; conductivity determines the vertical attenuation; and boundary matching converts the internal solution into a measurable surface impedance. The skin-depth relation provides the simplest physical interpretation of depth sensitivity, while the two-layer formulation demonstrates how finite layer thickness modifies the surface response. [1,7,21]

The approximation of negligible displacement current is appropriate for the low-frequency conducting-Earth regime considered here, but it should be checked when the frequency range or medium changes substantially. Likewise, $\mu = \mu_0$ is a useful approximation for ordinary crustal materials, whereas strongly magnetic materials may require a more general permeability. The largest conceptual limitation is the one-dimensional conductivity assumption. Horizontal contrasts, coastlines, faults, sedimentary basins and other geological structures can produce secondary fields that are not represented by a purely layered model. [1]

For micropulsation observations, these limitations suggest a staged interpretation. A one-dimensional layered model should first be used to establish the dominant depth-dependent response. Systematic residuals, frequency-dependent phase anomalies, or enhanced B_z near known lateral boundaries should then motivate a two-dimensional or three-dimensional analysis. This approach preserves the physical transparency of the layered model while avoiding overinterpretation of measurements affected by lateral conductivity contrasts. [1,9,19]

4. Conclusions

A theoretical framework has been presented for electromagnetic induction by geomagnetic micropulsations in a layered Earth. The principal conclusions are: (1) surface tangential electric and magnetic fields provide a direct measure of the electromagnetic impedance of the conducting Earth; (2) neglecting displacement current reduces the low-frequency induction problem to a diffusion-type vertical equation; (3) the skin depth $\delta = \sqrt{2/(\omega\mu\sigma)}$ demonstrates that long-period micropulsations

penetrate more deeply and are therefore valuable for vertical conductivity profiling; (4) a two-layer model relates the observed surface response to upper-layer conductivity, thickness, basement properties and source scale; and (5) strong horizontal conductivity discontinuities, especially coastlines, can generate an enhanced vertical magnetic component through lateral redistribution of induced currents. The combined framework provides a useful theoretical basis for interpreting geomagnetic micropulsation observations and for distinguishing depth-dependent induction from effects caused by lateral conductivity structure.

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