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SIMULTANEOUS FIVE SERIES EQUATIONS INVOLVING KONHAUSER BIORTHOGONAL POLYNOMIALS

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ABSTRACT:

Spencer and Fano [11] used the biorthogonal polynomials (for the case of k =2) in carrying out calculations involving penetration of gamma rays through matter. In the present paper an exact solution of simultaneous five series equations involving Konhauser – biorthogonal polynomials of first kind of different indices is obtained by multiplying factor technique due to Noble [8]. This technique has been modified by Thakare [13] to solve dual series equations involving orthogonal polynomials which led to disprove a possible conjecture of Askey [1] that dual series equations involving Jacobi polynomials of different indices cannot be solved. In this paper the solution of simultaneous five series equations involving generalized Laguerre polynomials also have been discussed in a particular case.

Keywords: ***Basic orthogonal polynomials, General Theory, Orthogonal functions and polynomials, Fractional derivatives and integrals, Laguree polynomials, Konhauser bi-orthogonal polynomials, Simultaneous five series equations.***

Mathematics Subject Classification: 33C45, 42C05, 33D45, 26A33.

1. INTRODUCTION

If we review the literature then we observe that the existing solutions on series equations are derived only from dual to triple series equations no further generalizations are available till date. Dual triple and quadruple series equations play an important role in finding the solution of mixed boundary value problems of elasticity, electrostatics and other fields of mathematical physics. Simultaneous dual and triple equations involving Konhauser biorthogonal polynomials have been consider by many authors [4] [9] .In this paper, we have consider Simultaneous Five Series Equations Involving Konhauser biorthogonal Polynomials which are extensions of dual and triple series equations considered by authors[4] [9] and we have obtained certain results. Konhauser [3] introduced a pair of sets of bi-orthogonal polynomials $Z_n^\alpha(x:k)$ and $Y_n^\alpha(x:k)$ with respect to the weight function x^αexp (-x) over the interval (0, ∞) based on the study of Preiser [10] . In fact Konhauser defined

$$Z_n^\alpha(x:k) = \frac{\Gamma(kn+\alpha+1)}{n!}\sum_{j=0}^{n}(-1)^j\binom{n}{j}\frac{x^{kj}}{\Gamma(kj+\alpha+1)}$$

And

$$Y_n^\alpha(x:k) = \frac{1}{n!}\sum_{p=0}^{n}\frac{x^p}{p!}\sum_{q=0}^{p}(-1)^q\binom{p}{q}\left[\frac{q+\alpha+1}{k}\right]_n$$

$Z_n^\alpha(x:k)$ is called Konhauser – biorthogonal set of the first kind and $Y_n^\alpha(x:k)$ the Konhauser – biorthogonal set of the second kind. For k=1, both the polynomials reduce to the generalized Laguerre polynomials $L_n^\alpha(x)$.

2. SIMULTANEOUS FIVE SERIES EQUATIONS

Simultaneous Five Series Equations Involving Konhauser Biorthogonal Polynomials are:

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\,\frac{A_{nj}}{\Gamma(kni+\alpha+p+1)}Z_{ni+p}^{\alpha}(x:k) = u_i(x)\,, \quad 0 \le x < a \qquad \textbf{(1.1)}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}Z_{ni+p}^{\beta+\delta-1}(x:k) = v_i(x)\,, \quad a < x < b \qquad \textbf{(1.2)}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}Z_{ni+p}^{\beta+\delta-1}(x:k) = w_i(x)\,, \quad b < x < c \qquad \textbf{(1.3)}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}Z_{ni+p}^{\beta+\delta-1}(x:k) = y_i(x)\,, \quad c < x < d \qquad \textbf{(1.4)}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}Z_{ni+p}^{\sigma}(x:k) = z_i(x)\,, \quad d < x < \infty \qquad \textbf{(1.5)}$$

Where $Z_n^\alpha(x:k)$is the Konhauser-biorthogonal polynomials,$u_i(x), v_i(x)w_i(x), y_i(x)\ and\ z_i(x)$ are prescribed functions, $a_{ij}\,, b_{ij}\,, and\ c_{ij}$ are known constants for $i = 1,2, \ldots\ldots s.$ and $j = 1,2, \ldots\ldots s. \beta + \delta + m > \alpha + 1 > 0$ and $+1 > \beta + \delta > 0$, m being some positive integer and p is a non- negative integer. A_{nj} $'s$ are the unknown constant for $j = 1,2, \ldots\ldots s.$in the series equations which are to be determine.

3. RESULTS USED IN THE SEQUEL

During the course of analysis the following results shall be needed:

(i) Biorthogonal relation was given by Konhauser [2] as follows:

$$\int_0^\infty e^{-x}\, x^\alpha Z_n^\alpha(x:k) Y_m^\alpha(x:k)dx = \frac{\Gamma(kn+\alpha+1)}{n!}\,\delta_m^n \qquad \textbf{(2.1)}$$

Where δ_m^n is Kranecker's delta.

(ii) Prabhakar [9] introduced the following m th differential form

$$\frac{d^m}{dx^m}[x^{\alpha+m} z_n^{\alpha+m}(x;k)] = \frac{\Gamma(kn+\alpha+m+1)}{\Gamma(kn+\alpha+1)} x^\alpha z_n^\alpha(x;k) \qquad \textbf{(2.2)}$$

with $\alpha > -1$.

(iii) Prabhakar [9] introduced the following fractional integrals, the first being the Riemann-Liouville fractional integral:

$$\int_0^\xi (\xi - x)^{\beta-1} x^\alpha Z_n^\alpha\,(x:k)dx = \frac{\Gamma(kn+\alpha+1)\Gamma(\beta)}{\Gamma(kn+\alpha+\beta+1)}\,\xi^{\alpha+\beta} z_n^{\alpha+\beta}\,(\xi;k) \quad \textbf{(2.3)}$$

When $\beta > 0, \alpha + \beta + 1 > 0$ and the second, the Weyl fractional integral

$$\int_\xi^\infty (x-\xi)^{\beta-1} e^{-x} Z_n^\alpha\,(x:k)dx = \Gamma(\beta)\,.\,e^{-\xi} Z_n^{\alpha-\beta}\,(\xi;k) \qquad \textbf{(2.4)}$$

Where $\alpha + 1 > \beta > 0$.

4. THE SOLUTION OF FIVE SERIES EQUATIONS

The solution of Simultaneous Five Series Equations Involving Konhauser Biorthogonal Polynomials is obtained by multiplying both the sides of the equation (1.1) by $x^\alpha(\xi - x)^{-\alpha+\beta+\delta+m-2}$ and integrating with respect to x over (0,ξ) and first fractional integral formula (2.3) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj} \ \Gamma(-\alpha+\beta+\delta+m-1)}{\Gamma(kni+\beta+p+\delta+m)} \xi^{\beta+\delta+m-1} Z_{ni+p}^{\beta+\delta+m-1}(\xi;k)$$

$$= \int_0^{\xi} x^{\alpha}(\xi-x)^{-\alpha+\beta+\delta+m-2} u_i(x)dx \qquad \textbf{(3.1)}$$

which can be written as

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} \frac{A_{nj}}{\Gamma(kni+\beta+p+\delta+m)} Z_{ni+p}^{\beta+\delta+m-1}(\xi;k)$$

$$= \frac{\xi^{-\beta-\delta-m+1}}{\Gamma(-\alpha+\beta+\delta+m-1)} \int_0^{\xi} x^{\alpha}(\xi-x)^{-\alpha+\beta+\delta+m-2} u_i(x)dx \qquad \textbf{(3.2)}$$

with $0 < x < \xi$ and $\beta+\delta+m > \alpha+1 > 0$.

Now multiplying both sides of equation (3.2) by $\xi^{\beta+\delta+m-1}$ and differentiating both sides 'm' times with respect to ξ and using the derivative formula (2.2) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} \frac{A_{nj}}{\Gamma(kni+\beta+p+\delta)} Z_{ni+p}^{\beta+\delta-1}(\xi;k) = \sum_{j=1}^{s} f_{ij} \frac{\xi^{-\beta-\delta+1}}{\Gamma(-\alpha+\beta+\delta+m-1)} \boldsymbol{U}_i(\xi)$$

$0 < \xi < a$ **(3.3)**

where $\boldsymbol{U}_i(\xi) = \frac{d^m}{dx^m} \int_0^{\xi} x^{\alpha}(\xi-x)^{-\alpha+\beta+\delta+m-2} u_i(x)dx$

$i = 1,2, \ldots\ldots s.$ **(3.4)**

and f_{ij} are the elements of the matrix $[b_{ij}][a_{ij}]^{-1}$.

Next multiplying both sides of equation (1.5) by $e^{-x}(x-\xi)^{\sigma-\beta-\delta}$ and integrate with respect to x over (ξ,∞) and using the second fractional integral formula (2.4) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij} \frac{A_{nj} \ \Gamma(\sigma-\beta-\delta+1)}{\Gamma(kni+\delta+p+\beta)} e^{-\xi} Z_{ni+p}^{\delta+\beta-1}(\xi:k) = \int_{\xi}^{\infty} e^{-x}(x-\xi)^{\sigma-\beta-\delta} z_i(x)dx \qquad \textbf{(3.5)}$$

Which can be written as,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\ \frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)} Z_{ni+p}^{\delta+\beta-1}(\xi{:}\,k)$$

$$= \frac{e^{\xi}}{\Gamma(\sigma-\beta-\delta+1)}\int_{\xi}^{\infty} e^{-x}(x-\xi)^{\sigma-\beta-\delta}\ z_i(x)dx \qquad \textbf{(3.6)}$$

Where $\xi < x < \infty$ and $\sigma+1 > \beta+\delta > 0$.

From equation (3.6), we get

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\ \frac{A_{nj}}{\Gamma(kni+\beta+p+\delta)} Z_{ni+p}^{\beta+\delta-1}(\xi;k) = \sum_{j=1}^{s} g_{ij}\ \frac{e^{\xi}}{\Gamma(\sigma-\beta-\delta+1)}\boldsymbol{Z}_i(\xi)$$

$$d < \xi < \infty \qquad \textbf{(3.7)}$$

where $\boldsymbol{Z}_i(\xi) = \int_{\xi}^{\infty} e^{-x}(x-\xi)^{\sigma-\delta-\beta}\ z_i(x)dx \ \ ; i = 1,2, \ldots\ldots s.$ **(3.8)**

and g_{ij} are the elements of the matrix $[b_{ij}\ ][c_{ij}\ ].^{-1}$

Now left hand sides of the equations (1.2),(1.3),(1.4),(3.3), (3.7) are identical. Applying the biorthogonal relation (2.1) of konhauser polynomials, we get the solution of the simultaneous five series equations in the form,

$$A_{nj} = \sum_{j=1}^{s} q_{ij}\ (ni+p)!\left[\sum_{j=1}^{s}\frac{f_{ij}}{\Gamma(-\alpha+\beta+\delta+m-1)}\int_0^a e^{-\xi}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)\boldsymbol{U}_i(\xi)d\xi\right.$$

$$+\int_a^b e^{-\xi}e^{\beta+\delta-1}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)v_{\boldsymbol{i}}(\xi)d\xi + \int_b^c e^{-\xi}e^{\beta+\delta-1}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)w_i(\xi)d\xi$$

$$\left.+\int_c^d e^{-\xi}e^{\beta+\delta-1}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)y_i(\xi)d\xi + \sum_{j=1}^{s}\frac{g_{ij}}{\Gamma(\sigma-\beta-\delta+1)}\int_d^{\infty} e^{\beta+\delta-1}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)\boldsymbol{Z}_i(\xi)d\xi\right]$$

Where q_{ij} are the elements of the matrix $[b_{ij}]^{-1}$ and$n = 0,1,2, \dots \dots$ and $j = 1,2, \dots . . s$. and $\boldsymbol{U}_i(\xi)$ and $\boldsymbol{z}_i(\xi)$ are defined by (3.4) and (3.8) respectively.

5. PARTICULAR CASE

It is very interesting for if we put k = 1 in equations (1.1), (1.2), (1.3), (1.4), (1.5) then Konhauser polynomials involved in these equations are reduced to generalized Laguerre polynomials and we receive the following simultaneous five series equations involving generalized Laguerre polynomials.

$$\frac{\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij} A_{nj}}{\Gamma(kni+\alpha+p+1)} L_{ni+p}^{\alpha}(x) = f_i(x), \quad 0 \le x < a \qquad \textbf{(4.1)}$$

$$\frac{\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} A_{nj}}{\Gamma(kni+\delta+p+\beta)} L_{ni+p}^{\beta+\delta-1}(x) = g_i(x), \quad a < x < b \qquad \textbf{(4.2)}$$

$$\frac{\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} A_{nj}}{\Gamma(kni+\delta+p+\beta)} L_{ni+p}^{\beta+\delta-1}(x) = h_i(x), \quad b < x < c \qquad \textbf{(4.3)}$$

$$\frac{\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij} A_{nj}}{\Gamma(kni+\delta+p+\beta)} L_{ni+p}^{\beta+\delta-1}(x) = j_i(x), \quad c < x < d \qquad \textbf{(4.4)}$$

$$\frac{\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij} A_{nj}}{\Gamma(kni+\delta+p+\beta)} L_{ni+p}^{\sigma}(x) = k_i(x), \quad d < x < \infty \qquad \textbf{(4.5)}$$

with the solution in the form,

$$A_{nj} = \sum_{j=1}^{s} q_{ij}\,(ni+p)! \left[\sum_{j=1}^{s} \frac{f_{ij}}{\Gamma(-\alpha+\beta+\delta+m-1)} \int_0^a e^{-\xi} L_{ni+p}^{\beta+\delta-1}(\xi;k)\boldsymbol{U}_i(\xi)d\xi\right.$$

$$+\int_a^b e^{-\xi} e^{\beta+\delta-1} L_{ni+p}^{\beta+\delta-1}(\xi;k)v_{\boldsymbol{i}}(\xi)d\xi + \int_b^c e^{-\xi} e^{\beta+\delta-1} L_{ni+p}^{\beta+\delta-1}(\xi;k)w_i(\xi)d\xi$$

$$+\int_c^d e^{-\xi} e^{\beta+\delta-1} L_{ni+p}^{\beta+\delta-1}(\xi;k) y_i(\xi) d\xi + \sum_{j=1}^{s} \frac{g_{ij}}{\Gamma(\sigma-\beta-\delta+1)} \int_d^\infty e^{\beta+\delta-1} L_{ni+p}^{\beta+\delta-1}(\xi;k) \boldsymbol{Z}_i(\xi) d\xi \Big]$$

Whereq_{ij} are the elements of the matrix $[b_{ij}]^{-1}$ and $n = 0,1,2, \dots \dots$ and $j = 1,2, \dots . . s$.and $\boldsymbol{U}_i(\xi)$ and $\boldsymbol{Z}_i(\xi)$ are defined by (3.4) and (3.8) respectively when k = 1.

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