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# SIMULTANEOUS QUADRUPLE SERIES EQUATIONS INVOLVING KONHAUSER BIORTHOGONAL POLYNOMIALS

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***ABSTRACT:***

***Spencer and Fano [11] used the biorthogonal polynomials (for the case of k = 2) in carrying out calculations involving penetration of gamma rays through matter. In the present paper an exact solution of simultaneous quadruple series equations involving Konhauser – biorthogonal polynomials of first kind of different indices is obtained by multiplying factor technique due to Noble [13]. This technique has been modified by Thakare [12] to solve dual series equations involving orthogonal polynomials which led to disprove a possible conjecture of Askey [6] that dual series equations involving Jacobi polynomials of different indices cannot be solved. In this paper the solution of simultaneous quadruple series equations involving generalized Laguerre polynomials also have been discussed in a particular case.***

***Keywords:*** ***Basic orthogonal polynomials, General Theory, Orthogonal functions and polynomials, Fractional derivatives and integrals, Laguree polynomials, Konhauser bi-orthogonal polynomials.***

**Mathematics Subject Classification:** 33C45, 42C05, 33D45, 26A33.

## 1. INTRODUCTION

Konhauser [8] introduced a pair of sets of bi-orthogonal polynomials $Z_n^\alpha(x:k)$ and $Y_n^\alpha(x:k)$ with respect to the weight function $x^\alpha$exp (-x) over the interval (0, ∞) based on the study of Preiser [10]. In fact Konhauser defined:

$$Z_n^\alpha(x:k) = \frac{\Gamma(kn+\alpha+1)}{n!}\sum_{j=0}^{n}(-1)^j\binom{n}{j}\frac{x^{kj}}{\Gamma(kj+\alpha+1)}$$

And

$$Y_n^\alpha(x:k) = \frac{1}{n!}\sum_{p=0}^{n}\frac{x^p}{p!}\sum_{q=0}^{p}(-1)^q\binom{p}{q}\left[\frac{q+\alpha+1}{k}\right]_n$$

$Z_n^{\alpha}(x:k)$ is called Konhauser – biorthogonal set of the first kind and $Y_n^{\alpha}(x:k)$ the Konhauser – biorthogonal set of the second kind. For k =1, both the polynomials reduce to the generalized Laguerre polynomials $L_n^{\alpha}(x)$.

## 2. SIMULTANEOUS QUADRUPLE SERIES EQUATIONS

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\,\frac{A_{nj}}{\Gamma(kni+\alpha+p+1)}Z_{ni+p}^{\alpha}(x:k) = u_i(x)\,, \quad 0 \le x < a \tag{1.1}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}Z_{ni+p}^{\beta+\delta-1}(x:k) = v_i(x)\,, \quad a < x < b \tag{1.2}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}Z_{ni+p}^{\beta+\delta-1}(x:k) = w_i(x)\,, \quad b < x < c \tag{1.3}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}Z_{ni+p}^{\sigma}(x:k) = y_i(x)\,, \quad c < x < \infty \tag{1.4}$$

Where $Z_n^{\alpha}(x:k)$ is the Konhauser- biorthogonal polynomials, $u_i(x), v_i(x)$ $w_i(x)$ $and$ $y_i(x)$ are prescribed functions, $a_{ij}, b_{ij}, and\ c_{ij}$ are known constants for $i = 1,2, \ldots\ldots s.$ and $j = 1,2, \ldots\ldots s.$ $\beta + \delta + m > \alpha + 1 > 0$ and $\sigma + 1 > \beta + \delta > 0$ , m being some positive integer and p is a non- negative integer. $A_{nj}'s$ are the unknown constant for $j = 1,2, \ldots\ldots s.$ in the series equations which are to be determine.

## 3. RESULTS USED IN THE SEQUEL

During the course of analysis the following results shall be needed:

(i) Biorthogonal relation was given by Konhauser [7 ] as follows:

$$\int_0^{\infty} e^{-x}\, x^{\alpha} Z_n^{\alpha}(x:k) Y_m^{\alpha}(x:k)dx = \frac{\Gamma(kn+\alpha+1)}{n!}\,\delta_m^n \tag{2.1}$$

where $\delta_m^n$ is Kranecker's delta.

(ii) Prabhakar [9 ] introduced the following m th differential form:

$$\frac{d^m}{dx^m}\left[x^{\alpha+m} z_n^{\alpha+m}(x;k)\right] = \frac{\Gamma(kn+\alpha+m+1)}{\Gamma(kn+\alpha+1)}\,x^{\alpha} z_n^{\alpha}(x;k) \tag{2.2}$$

with $\alpha > -1$.

(iii) Prabhakar [ 9] introduced the following fractional integrals, the first being the Riemann-Liouville fractional integral:

$$\int_0^{\xi}(\xi - x)^{\beta-1}x^{\alpha}Z_n^{\alpha}(x:k)dx = \frac{\Gamma(kn+\alpha+1)\Gamma(\beta)}{\Gamma(kn+\alpha+\beta+1)}\xi^{\alpha+\beta}Z_n^{\alpha+\beta}(\xi;k) \quad \textbf{(2.3)}$$

when $\beta > 0, \alpha + \beta + 1 > 0$ and the second, the Weyl fractional integral

$$\int_{\xi}^{\infty}(x - \xi)^{\beta-1}e^{-x}Z_n^{\alpha}(x:k)dx = \Gamma(\beta)\,.\,e^{-\xi}Z_n^{\alpha-\beta}(\xi;k) \quad \textbf{(2.4)}$$

where $\alpha + 1 > \beta > 0$.

## 4. THE SOLUTION OF QUADERUPLE SERIES EQUATIONS

Multiplying both the sides of the equation (1.1) by $x^{\alpha}(\xi - x)^{-\alpha+\beta+\delta+m-2}$ and integrating with respect to x over (0, $\xi$) and first fractional integral formula (2.3) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}\ \Gamma(-\alpha+\beta+\delta+m-1)}{\Gamma(kni+\beta+p+\delta+m)}\xi^{\beta+\delta+m-1}Z_{ni+p}^{\beta+\delta+m-1}(\xi;k)$$

$$= \int_0^{\xi} x^{\alpha}(\xi - x)^{-\alpha+\beta+\delta+m-2}u_i(x)dx \quad \textbf{(3.1)}$$

which can be written as

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\frac{A_{nj}}{\Gamma(kni+\beta+p+\delta+m)}Z_{ni+p}^{\beta+\delta+m-1}(\xi;k)$$

$$= \frac{\xi^{-\beta-\delta-m+1}}{\Gamma(-\alpha+\beta+\delta+m-1)}\int_0^{\xi} x^{\alpha}(\xi - x)^{-\alpha+\beta+\delta+m-2}u_i(x)dx \quad \textbf{(3.2)}$$

with $0 < x < \xi$ and $\beta + \delta + m > \alpha + 1 > 0$.

Now multiplying both sides of equation (3.2) by $\xi^{\beta+\delta+m-1}$ and differentiating both sides 'm' times with respect to $\xi$ and using the derivative formula (2.2) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\beta+p+\delta)} Z_{ni+p}^{\beta+\delta-1}(\xi;k) = \sum_{j=1}^{s} f_{ij}\,\frac{\xi^{-\beta-\delta+1}}{\Gamma(-\alpha+\beta+\delta+m-1)}\boldsymbol{U}_i(\xi)$$

$$0<\xi<a \tag{3.3}$$

where $\boldsymbol{U}_i(\xi) = \frac{d^m}{dx^m}\int_0^{\xi} x^{\alpha}(\xi-x)^{-\alpha+\beta+\delta+m-2}\,u_i(x)dx$

$$i = 1,2,\ldots..s. \tag{3.4}$$

and $f_{ij}$ are the elements of the matrix $[b_{ij}][a_{ij}]^{-1}$.

Next multiplying both sides of equation (1.4) by $e^{-x}(x-\xi)^{\sigma-\beta-\delta}$ and integrate with respect to x over $(\xi,\infty)$ and using the second fractional integral formula (2.4) we get,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\,\frac{A_{nj}\Gamma(\sigma-\beta-\delta+1)}{\Gamma(kni+\delta+p+\beta)} e^{-\xi} Z_{ni+p}^{\delta+\beta-1}(\xi:k) = \int_{\xi}^{\infty} e^{-x}(x-\xi)^{\sigma-\beta-\delta}\,z_i(x)dx \tag{3.5}$$

which can be written as,

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)} Z_{ni+p}^{\delta+\beta-1}(\xi:k)$$

$$= \frac{e^{\xi}}{\Gamma(\sigma-\beta-\delta+1)}\int_{\xi}^{\infty} e^{-x}(x-\xi)^{\sigma-\beta-\delta}\,z_i(x)dx \tag{3.6}$$

Where $\xi<x<\infty$ and $\sigma+1>\beta+\delta>0$.

From equation (3.6), we get

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\beta+p+\delta)} Z_{ni+p}^{\beta+\delta-1}(\xi;k) = \sum_{j=1}^{s} g_{ij}\,\frac{e^{\xi}}{\Gamma(\sigma-\beta-\delta+1)}\boldsymbol{Z}_i(\xi) \qquad c<\xi<\infty \tag{3.7}$$

where $\boldsymbol{Z}_i(\xi) = \int_{\xi}^{\infty} e^{-x}(x-\xi)^{\sigma-\delta-\beta}\,z_i(x)dx \;\; ; i = 1,2,\ldots..s.$ **(3.8)**

and $g_{ij}$ are the elements of the matrix $[b_{ij}][c_{ij}]^{-1}$.

Now left hand sides of the equations (1.2), (1.3), (3.3), (3.7) are identical. Applying the biorthogonal relation (2.1) of konhauser polynomials, we get the solution of the simultaneous quadruple series equations in the form,

$$A_{nj} = \sum_{j=1}^{s} q_{ij}\,(ni+p)!\left[\sum_{j=1}^{s} \frac{f_{ij}}{\Gamma(-\alpha+\beta+\delta+m-1)} \int_{0}^{a} e^{-\xi}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)\boldsymbol{U}_i(\xi)d\xi\right.$$

$$+\int_a^b e^{-\xi}e^{\beta+\delta-1}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)v_{\boldsymbol{i}}(\xi)d\xi + \int_b^c e^{-\xi}e^{\beta+\delta-1}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)w_i(\xi)d\xi$$

$$\left.+\sum_{j=1}^{s}\frac{g_{ij}}{\Gamma(\sigma-\beta-\delta+1)}\int_c^{\infty} e^{\beta+\delta-1}\gamma_{ni+p}^{\beta+\delta-1}(\xi;k)\boldsymbol{Z}_i(\xi)d\xi\right]$$

Where $q_{ij}$are the elements of the matrix $[b_{ij}]^{-1}$ and $n = 0,1,2,\ldots\ldots..$ and $j = 1,2,\ldots..s.$ and $\boldsymbol{U}_i(\xi)$ and $\boldsymbol{Z}_i(\xi)$ are defined by (3.4) and (3.8) respectively.

## 5. PARTICULAR CASE

It is very interesting for if we put k = 1 in equations (1.1), (1.2), (1.3), (1.4) then Konhauser polynomials involved in these equations are reduced to generalized Laguerre polynomials and we receive the following simultaneous quadruple series equations involving generalized Laguerre polynomials.

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} a_{ij}\,\frac{A_{nj}}{\Gamma(kni+\alpha+p+1)}L_{ni+p}^{\alpha}(x) = f_i(x)\,, \qquad 0 \le x < a \qquad \textbf{(4.1)}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}L_{ni+p}^{\beta+\delta-1}(x) = g_i(x)\,, \qquad a < x < b \qquad \textbf{(4.2)}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} b_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}L_{ni+p}^{\beta+\delta-1}(x) = h_i(x)\,, \qquad b < x < c \qquad \textbf{(4.3)}$$

$$\sum_{n=0}^{\infty}\sum_{j=1}^{s} c_{ij}\,\frac{A_{nj}}{\Gamma(kni+\delta+p+\beta)}L_{ni+p}^{\sigma}(x) = j_i(x)\,, \qquad c < x < \infty \qquad \textbf{(4.4)}$$

with the solution in the form,

$$A_{nj} = \sum_{j=1}^{s} q_{ij}\,(ni+p)!\left[\sum_{j=1}^{s} \frac{f_{ij}}{\Gamma(-\alpha+\beta+\delta+m-1)} \int_{0}^{a} e^{-\xi} L_{ni+p}^{\beta+\delta-1}(\xi;k)\boldsymbol{U}_i(\xi)d\xi\right.$$

$$+\int_a^b e^{-\xi} e^{\beta+\delta-1} L_{ni+p}^{\beta+\delta-1}(\xi;k)v_i(\xi)d\xi + \int_b^c e^{-\xi} e^{\beta+\delta-1} L_{ni+p}^{\beta+\delta-1}(\xi;k)w_i(\xi)d\xi$$

$$\left.+\sum_{j=1}^{s} \frac{g_{ij}}{\Gamma(\sigma-\beta-\delta+1)} \int_c^{\infty} e^{\beta+\delta-1} L_{ni+p}^{\beta+\delta-1}(\xi;k)\boldsymbol{Z}_i(\xi)d\xi\right]$$

where $q_{ij}$ are the elements of the matrix $[b_{ij}]^{-1}$ and $n = 0,1,2,\dots\dots$ and $j = 1,2,\dots\dots s.$ and $\boldsymbol{U}_i(\xi)$ and $\boldsymbol{Z}_i(\xi)$ are defined by (3.4) and (3.8) respectively when k = 1.

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