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A SYSTEM OF SEMILINEAR ELLIPTIC NEUMANN BOUNDARY VALUE PROBLEM

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Abstract: - ***In this paper we use the maximum principles to study symmetry properties of solutions for system of semilinear elliptic boundary value problem with Neumann condition, to the elliptic operators more general than the Laplacian operator on the unit ball in n dimensional Euclidean space with n ≥3.***

Keywords: ***Maximum principle; Radial symmetry; System of nonlinear elliptic Boundary value problem, Neumann boundary condition.***

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1. INTRODUCTION

To study the behavior of solutions of system of elliptic boundary value problems with Neumann boundary conditions is an interesting problem in the point of view of differential geometry and elliptic boundary value problems. The problem of the interest is

$$\int u = f_1 (v); \quad \text{in } B$$

$$\int v = f_2 (u) \quad \text{in } B$$

$$\partial u / \partial \eta = g_1 (v) \text{ on } \partial B$$

$$\partial v / \partial \eta = g_2 (u) \text{ on } \partial B.$$

Where B is ball of unit radius in R^n,

$\partial / \partial \eta$ denote the outer normal derivative to ∂B and f_1; f_2; g_1 and g_2 are functions defined in **R**.

The maximum principle is one of the most used tools in the study of some elliptic boundary value problems. A version of maximum principle allows us to compare locally surfaces that coincide at a point. Also the maximum principle and Alexandrov reflection principle have been used to prove symmetry of solutions of boundary value problems with respect to some point, some plane, and symmetry of domains by the authors

Serrin [14], Gidas, Ni and Nirenberg [4, 5], Berestycki and Nirenberg [2]. Castro, Maya and Shivaji [3] summarized the recent developments of semipositone problems.

Reichel [13] obtained the radial symmetry for the domain and solutions of

$$\int u + f (u; |\nabla u|) = 0$$

on exterior domain subject to the over determined boundary conditions

$$\partial u / \partial v = const; \quad u > 0$$

on the boundary, by using Alexandro_-Serrin method of moving hyperplanes. Bensedik and Bouchekif [1] proved the symmetry result of positive solutions for a Neumann boundary value problem,

$$L(u) = -u'' + q^2 u = |u|^P f(x); \quad x \ \varepsilon \ (a; b)$$

$$u'(a) = u'(b)$$

making generalization of the work of Padro J. Toress, [15]. In 2012, Marin, Ortiz and Rodriguez [9, 10] proved the symmetry of Neumann boundary value problem.

In this paper we prove that solution of the system of elliptic boundary value problem with Neumann condition,

$$a \int u + b \ \Sigma^n{}_{i=1} \ u_i = f_1(v)$$

$$a \int v + b \ \Sigma^n{}_{i=1} \ v_i = f_2(u)$$

$$\partial u / \partial \eta = g_1(v) ;$$

$$\partial v / \partial \eta = g_2(u)$$

are radially symmetric with respect to the origin, where a : **B** $\rightarrow$ R is a bounded function and symmetric with respect to the origin such that $a(x) > 0$ for all x ε **B** and b : **B** $\rightarrow$ R is a bounded and odd function. Let **η** denote the normal vector to ∂B and f_1; f_2; g_1 and g_2 ε C (**R**). Furthermore suppose that f_1; f_2 are strictly increasing and g_1 , g_2 are strictly decreasing.

Our proof shows that the techniques used in [2 , 4, 7, 14] for study of symmetric solutions of the ellipotic boundary value problem with Dirichlet condition can be applied to prove symmetry of solutions of the ellipotic boundary value problem with Neumann condition. Recently author Patil [6] proved the symmetry result for the Neumann boundary value problem, $-u'' + q^2 u = u^2 x^2$ with boundary condition $u'(-1) = u'(1)$ in the interval (-1 , 1).

2. MAIN RESULT

Now we state the main theorem for symmetry of solutions of system of elliptic Neumann boundary value problem.

THEOREM 2.1

Let **(u, v)** ε $C^2(B) \cup C^0(B)$ be a positive solution of the elliptic boundary value problem with Neumann condition.

$$a \int u + b\iota\Sigma^n{}_{i=1} \ u_i = f_1(v)$$

$$a \int v + b\Sigma^n{}_{i=1} \ v_i = f_2(u) \qquad (2.1)$$

$$\partial u / \partial \eta = g_1 (v) ;$$

$$\partial v / \partial \eta = g_2 (u)$$

Where a ,b, f_1; f_2; g_1 and g_2 are as defined in previous section. Then the solution (u; v) must be radially symmetric with respect to the origin.

Proof: Let x ε **B** and x^0 denotes the reflection of x with respect to the hyperplane $x_n = 0$.

Then x and x' both are in B.

∴ System of equations in 2.1 is satisfied by both x and x'.

$a \int u(x) + b\, \Sigma^{n}{}_{i=1}\ u_i(x) = f_1(v(x))$ in B (2.2)

$a \int v(x) + b\, \Sigma^{n}{}_{i=1}\ v_i(x) = f_2(u(x))$ in B (2.2)

$\partial u(x) / \partial \boldsymbol{\eta} = g_1(v(x))$ on ∂B (2.3)

$\partial v(x) / \partial \boldsymbol{\eta} = g_2(u(x))$ on ∂B (2.4)

x' also satisfies the same equations that x does.

$a \int u(x') + b\ \Sigma^{n}{}_{i=1}\ u_i\ (x') = f_1(v(x'))$ in B (2.6)

$a \int v(x') + b\, \Sigma^{n}{}_{i=1}\ v_i(x') = f_2(u(x'))$ in B (2.7)

$\partial u(x') / \partial \boldsymbol{\eta} = g_1(v(x'))$ on ∂B (2.8)

$\partial v(x') / \partial \boldsymbol{\eta} = g_2(u(x'))$ on ∂B (2.9)

We define the function w_1 and w_2 as follows,

$w_1 = u(x) – u(x')$ (2.10)

$w_2 = v(x) – v(x')$ (2.11)

In consequence w_1 and w_2 satisfies

$a \int w_1\ (x) + b\, \Sigma^{n}{}_{i=1}\ w_{1i}\ (x) = f_1(v(x)) - f_1(v(x'))$ in B (2.12)

$a \int w_2\ (x) + b\, \Sigma^{n}{}_{i=1}\ w_{2i}\ (x) = f_2(v(x)) – f_2(v(x'))$ in B (2.13)

$\partial w_1\ (x) / \partial \boldsymbol{\eta} = g_1(v(x)) - g_1(v(x'))$ on ∂B (2.14)

$\partial w_2\ (x) / \partial \boldsymbol{\eta} = g_2(u(x)) - g_2(u(x'))$ on ∂B (2.15)

$w_1 = 0\ ,\ w_1 = 0$ in $\mathbf{B} \cup \{x_n = 0\}$ (2.16)

Since w_1 and w_2 are continuous in **B** , there exist points x_m and x_M such that

$w_1\ (x_m) = \min_{\mathbf{B}} \mathbf{(w_1)}$

$w_1\ (x_M) = \max_{\mathbf{B}} \mathbf{(w_1)}$

$w_2\ (x_m) = \min_{\mathbf{B}} \mathbf{(w_2)}$

$w_2\ (x_M) = \max_{\mathbf{B}} \mathbf{(w_2)}$

We claim that the points x_m and x_M , cannot be in ∂B.

Suppose x_M ε ∂ B and satisfies

$w_1\ (x_M) > w_1(x),\ w_2\ (x_M) > w_2(x)$ for all x ε B.

Therefore

$\partial\ w_1(\ x_M) / \partial\eta\ \ > 0$

$\partial\ w_2(\ x_M) / \partial\eta\ \ > 0$

And

$w_1(\ x_M)\ \ > 0$

$w_2(\ x_M)\ \ > 0$

In consequence

$g_2\ (\ u\ (x_M)) - g_2\ (\ u\ (x'_M))\ \rho\ 0$

$g_1\ (\ v\ (x_M)) - g_1\ (\ v\ (x'_M))\ \rho\ 0$

Since g_1 and g_2 are strictly decreasing, we have,

$u\ (x_M)\ \beta\ \ u\ (x'_M)$ and $v\ (x_M)\ \beta\ \ v\ (x'_M)$

Then $w_1(x_M)\ \beta\ 0$ and $w_1(x_M)\ \beta\ 0$

Which contradicts to w1 $(x_M) > 0$ and $w_2\ (x_M) > 0$

.Thus, x_M cannot be in ∂B.

Suppose x_m ε ∂ B and satisfies

$w_1\ (x_m) < w_1(x),\ w_2\ (x_m) < w_2(x)$ for all x ε B

Therefore,

$\partial\ w_1\ (\ x_m) / \partial\eta\ \ < 0$

$\partial\ w_2\ (\ x_m) / \partial\eta\ \ < 0$

And

$w_1\ (\ x_m)\ \ < 0$

$w_2\ (\ x_m)\ \ < 0$

In consequence

$g_2\ (u\ (x_m)) - g_2\ (\ u\ (x'_m))\ \beta\ 0$

$g_1\ (v\ (x_m)) - g_1\ (\ v\ (x'_m))\ \beta 0$

Since g_1 and g_2 are strictly decreasing, we have,

$u(x_m)\ \rho\ u(x'_m)$ and $v(x_m)\ \rho\ v(x'_m)$

Then $w_1(x_m)\ \rho\ 0$ and $w1(x_m)\ \rho\ 0$

Which contradicts to $w1(x_m) < 0$ and $w_2(x_m) < 0$

.Thus, x_m cannot be in ∂B.

Hence x_M , x_m ε B.

So,

$$a(x_m) \int w_1 x_m + b(x_m) \Sigma_{i=1}^{n} w_{1i} (x_m)\ \rho\ 0$$

$$a(x_m) \int w_2 x_m + b(x_m) \Sigma_{i=1}^{n} w_{2i} (x_m)\ \rho\ 0$$

$$a(x_M) \int w_1 x_M + b(x_M) \Sigma_{i=1}^{n} w_{1i} (x_M)\ \beta\ 0$$

$$a(x_M) \int w_2 x_M + b(x_M) \Sigma_{i=1}^{n} w_{2i} (x_M)\ \beta\ 0$$

Since f_1 and f_2 are strictly increasing we conclude that,

$$u(x_m)\ \rho\ u(x'_m)$$

$$u(x_M)\ \beta\ u(x'_M)$$

$$v(x_m)\ \rho\ v(x'_m)$$

$$v(x_M)\ \rho\ v(x'_M)$$

Then

$$w_1(x_m) = w_1(x_M) = 0 \quad \text{and}$$

$$w_2(x_m) = w_2(x_M) = 0$$

Hence, $w_1 = 0$; $w2 = 0$ in **B**, So (u , v) is symmetric with respect to the plane $x_n = 0$.

Since B is unit ball and we can apply the same arguments in all directions, it follows that u is radially symmetric with respect to the origin.

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