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Partial vector η convexity non-differentiable Multi-objective Fractional Minimax Problem

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ABSTRACT:

In this paper we introduce the concepts of partial vector-ρ _ η convexity and its generalizations for a vector valued function. We establish sufficient optimality conditions and duality results for generalized minimax Multi-objective fractional programming problem involving locally Lipschitz functions under non differentiable conditions. Moreover, the main feature of the present work is to point out that it is not necessary to impose vector-ρ - η convexity vector-ρ - η convexity assumptions on all components of vector valued functions involved in the problem to develop duality results. It is sufficient to impose these conditions only on few components. In this section, with the help of (EP_v), we introduce dual (FD) to the problem (FP) and under suitable vector-ρ - η convexity assumptions and also duality results relating to (EP_v) and (FD). To establish the optimality conditions and duality, we shall make use of problem (FP_v).

1. INTRODUCTION:

In mathematical programming we are faced with the problem of finding optimality criteria these criteria may be necessary/sufficient conditions. Bazara et al. [14] obtain optimality criteria without differentiability. Hanson [41] introduced the concept of invexity as broad generalizations of convexity. Jeyakumar [49] introduced ρ -Invex functions and studied various results for a single objective nonlinear programming problem. Mond and Jeyakumar [48] have introduced the notion of V-invexity for a vector function. Recently, Bector et al. [20] developed sufficient optimality conditions and established duality results under V-invexity type of assumptions on the objective and constraint functions. They worked under differentiability assumptions. Bhatia and Garg [13] introduced V-ρ -Invexity for non smooth functions and established duality results for multi-objective programming problem. Davinder Bhatia and Hitesh Arora [24] introduced partial vector invexity in fractional minimax programming problem. But no serious attempt made in utilizing the recent developed concepts like vector η convexity and vector-ρ - η convexity for non smooth functions in multi-objective fractional programming problem. Hence in this chapter an attempt is made to fill the gap in this aim of research by developing some theorems and methods to solve vector η convexity and vector-ρ - η convexity for non smooth functions in multi-objective fractional programming problem.

In this paper, we introduce the concepts of partial vector-ρ _ η convexity and its generalizations for a vector valued function. We establish sufficient optimality conditions and duality results for generalized minimax Multiobjective fractional programming problem involving locally Lipschitz functions under non differentiable conditions. Moreover, the main feature of the present work is to point out that it is not necessary to impose vector-ρ - η convexity vector-ρ - η convexity assumptions on all components of vector valued functions involved in the problem to develop duality results. It is sufficient to impose these conditions only on few components.

2. DEFINITIONS:

The following definitions are used further discussion.

2.1. Definition: If $\theta : X \rightarrow R$ Lipschitz on X, the generalized directional derivative of θ at $x \in X$ in the direction

of $\eta(x, x^*) \in R^n$, denoted by $\theta^*(x\,;\,\eta(x, x^*))$ is given by

$$\theta^*(x\,;\,\eta(x, x^*)) = \underset{\lambda \downarrow 0}{\mathrm{Lt}} \ \underset{y \rightarrow x^*}{\mathrm{Sup}} \left[\frac{\theta(y + \lambda \eta(x, x^*)) - \theta(y)}{\lambda} \right].$$

2.2. Definition: The generalized η - gradient of θ at $x \in X$ in the direction of $\eta(x,x^*)$ denoted by the set $\partial_n \theta(x)$ and is defined as follows

$$\partial_n \theta(x) = \{ \xi \in R^n : \theta^*(x\,;\,\eta(x,x^*)) \geq \xi^T \eta(x, x^*), \forall x \in X \}$$

When θ is smooth (continuously differentiable), $\partial_n \theta(x)$ coincides with sub- differential of convex functions. For the sake of convenience we denote ∂_n by ∂ and let K= {1,2,….k} and $K^1 \subseteq K$.

2.3. Definition: A vector function $\frac{f_i(x)}{g_i(x)}: X \to R^k$, locally Lipschitz at $u \in X$ is said to be vector-ρ - η convexity at u if there exist functions $\eta, \varphi : XxX \to R^n$, a real number ρ and $\theta_i : X \times X \to R^+ \setminus \{0\}, i \in K^1$, such that for all $x \in X$

$$\frac{f_i(x)}{g_i(x)} - \frac{f_i(u)}{g_i(u)} \geq \theta_i(x,u)\xi^T \eta(x,u) + \rho \|\varphi(x,u)\|^2 \ \forall \xi_i \in \partial f_i(u),\ i \in K^1$$

2.4. Definition: A vector function $\frac{f_i(x)}{g_i(x)}: X \to R^k$, locally Lipschitz at $u \in X$ is said to be vector-ρ - η pseudo convex at u if there exist functions $\eta, \varphi : XxX \to R^n$, a real number ρ and $\theta_i : X \times X \to R^+ \setminus \{0\}, i \in K^1$, such that for all $x \in X$

$$\xi^T \eta(x,u) + \rho \|\varphi(x,u)\|^2 \geq 0 \Rightarrow \sum_{i \in K^1} \theta_i(x,u) f_i(x) \geq \sum_{i \in K^1} \theta_i(x,u) f_i(u) \ \forall \xi_i \in \partial f_i(u),\ i \in K^1.$$

2.2.5. Definition: A vector function $\frac{f_i(x)}{g_i(x)}: X \to R^k$, locally Lipschitz at $u \in X$ is said to be vector-ρ - η – quasi convex at u if there exist functions $\eta, \varphi : XxX$ 敺 R^n, a real number ρ and $\theta_i : X \times X \to R^+ \setminus \{0\}, i \in K^1$, such that for all $x \in X$

$$\sum_{i \in K^1} \theta_i(x,u) f_i(x) \leq \sum_{i \in K^1} \theta_i(x,u) f_i(u) \Rightarrow \xi^T \eta(x,u) \leq -\rho \|\varphi(x,u)\|^2 \quad \forall \xi_i \in \partial f_i(u),\ i \in K^1.$$

3. FORMULATIONS:

3.1. We now consider the following minimax multi-objective fractional programming problem as the primal problem:

$$\text{(FP)} \qquad v = \min_{x\in X} \max_{1\le i\le k} \frac{f_i(x)}{g_i(x)},$$

Subject to $\quad h_j(x) \le 0, \; j = 1,2,\ldots,m \qquad (1)$

where $f_i, g_i : R^n \to R, \; i = 1,2,..,k$ and $h_j : R^n \to R, \; j = 1,2,\ldots,m$ are real valued functions and $X \subset R^n$ .Also $f_i, g_i, \; i = 1,2,..,k$ and $h_j, \; j = 1,2,\ldots,m$ are locally Lipschitz functions around a point of X.

For each $i = 1,2,..,p$, let $f_i(x) \ge 0$ and $g_i(x) > 0$ for all x in X.

3. 2. Optimality Conditions

Considered the following minimax nonlinear parametric programming problem in the parameter v:

$$(FP_v) \qquad F_i(v) = \min_{x\in X} \max_{1\le i\le k} [f_i(x) - vg_i(x)] \qquad (2)$$

To establish the optimality conditions and duality, we shall make use of problem (FP_v).

3.3. We now have the following programming problem that is equivalent to (EP_v) for a given v:

$$(EP_v) \qquad \text{Min } q, \qquad (3)$$

$$\text{subject to } [f_i(x) - vg_i(x)] \le q, \; i = 1,2,..,k, \qquad (4)$$

$$h_j(x) \le 0, \; j = 1,2,\ldots,m, \qquad (5)$$

were $q \in R$ and $x,v \in X$.

3.4. Dual problem Formulation:

In this section, with the help of (EP_v), we introduce dual (FD) to the problem (FP) and under suitable vector-ρ - η convexity assumptions and also duality results relating to (EP_v) and (FD). Mond-Weir [67] type dual (FD) to the equivalent problem (EP_v) is stated as follows:

(FD) Maximize y

$$\text{subject to} \quad 0 \in \{ \sum_{i=1}^{k} \lambda_i \partial\left(f_i(u) - vg_i(u)\right) + \sum_{j=1}^{m} \mu_j \partial h_j(u) \}, \qquad (6)$$

$$\lambda_i \{\left(f_i(u) - vg_i(u)\right) - y\} = 0, \; i = 1,2,..,k, \qquad (7)$$

$$\mu_j \, h_j(u) = 0, \quad j = 1,2,..,m, \qquad (8)$$

$$\sum_{i=1}^{k} \lambda_i^* = 1, \qquad (9)$$

$$u \in X, y \in R, \lambda_i^* \in R_+{}^k, \mu_j^* \in R_+{}^m. \qquad (10)$$

Let T and W denotes the set of all feasible solutions of (EP_v) and (FD) respectively. We now establish the following duality theorems relating to (EP_v) and (FD).

4. LEMMA:

4.1. Lemma: If (FP) has an optimal solution x_{v^*}, hence after to be denoted by x*, with optimal value of the (FP) – objective as v*, then $F_i(v^*) = 0$. Conversely, if $F_i(v^*) = 0$, then (FP) and (FP_{v^*}) have the same optimal solution set.

4.2. Lemma: If (x,v,q) is (EP_v) – feasible, then x is (FP)- feasible. If x is (FP) – feasible, then there exist v and q such that (x, v, q) is (EP_v) –feasible.

4.3. Lemma: x* is (FP) – optimal with corresponding optimal value of the (FP) – objective equal to v* iff (x*,v*,q*) is (EP_v) optimal with corresponding optimal value of the (EP_v) –objective equal to zero, i.e., q* = 0.

5. NECESSARY AND SUFFICIENT OPTIMALITY CONDITIONS

5.1. Necessary Theorem x* be an optimal solution of (FP) with optimal value of the (FP) – objective equal to v*. Let an appropriate constraint qualification hold for (EP_{v*}), then there exist $q^* \in R, \lambda_i^* \in R^k, \mu_j^* \in R^m$ such that $(x^*, v^*, \lambda^*, \mu^*)$ satisfies

$$0 \in \{\sum_{i=1}^{k} \lambda_i^* \partial\left(f_i(x^*) - v^* g_i(x^*)\right) + \sum_{j=1}^{m} \mu_j^* \partial h_j(x^*)\}, \qquad (11)$$

$$\lambda_i^*\left(f_i(x^*) - v^* g_i(x^*)\right) - q^* = 0, \; i = 1,2,..,k, \qquad (12)$$

$$\mu_j^* h_j(x^*) = 0, \quad j = 1,2,..,m, \qquad (13)$$

$$\left(f_i(x^*) - v^* g_i(x^*)\right) \le q^*, \; i = 1,2,..,k, \qquad (14)$$

$$h_j(x) \le 0, \; j = 1,2,\ldots,m \qquad (15)$$

$$\sum_{i=1}^{k} \lambda_i^* = 1, \qquad (16)$$

$$q^* = 0, \qquad (17)$$

$$q^* \in R, \lambda_i^* \in R^k, \mu_j^* \in R^m, \; \lambda_i^* \ge 0, \mu_j^* \ge 0. \qquad (18)$$

Proof: It Follows directly by writing the necessary optimality conditions to the problem (EP_{v*}) there exists $\lambda_i^* \in R^k, \mu_j^* \in R^m$ such that the following conditions hold:

$$0 \in \{\sum_{i=1}^{k} \lambda_i^* \partial\left(f_i(x^*) - v^* g_i(x^*)\right) + \sum_{j=1}^{m} \mu_j^* \partial h_j(x^*)\},$$

$$\mu_j^* h_j(x^*) = 0, \quad j= 1,2,..,m,$$

$$\lambda_i^* \geq 0, \ i= 1,2,\ldots,k,$$

$$\mu_j^* \geq 0, \ j= 1,2,\ldots.,m.$$

Where

$$V^* = \frac{f_i(x^*)}{g_i(x^*)}, \ i= 1,2,\ldots,k,$$

setting

$$\lambda_i^* = \frac{\lambda_i}{\sum_{i=1}^{k} \lambda_i}, \ i= 1,2,\ldots,k,$$

$$\mu_j^* = \frac{\mu_j}{\sum_{i=1}^{k} \lambda_i}, \ j= 1,2,\ldots.,m,$$

we obtain that the conditions 11 to18 holds.

Which completes proof of the theorem.

5.2. Sufficient Optimality theorem:

Let x* be a feasible solution of (FP). Assume that there exist $\lambda_i^* \in R_+^k$, with $\sum_{i=1}^{k} \lambda_i^*$, $\mu_j^* \in R_+^m$ such that (11)-(18) are satisfied. Further, let

(a) $(f_i(x^*) - v^* g_i(x^*)), i \in I(x^*)$ be vector-ρ - η pseudocnvex with respect to η.

(b) $h_j(x), j \in J(x^*)$ vector-ρ - η - quasi convex with respect to η and (c) $(\rho + \rho^1) > 0$,

where, $I(x^*) = \{ i \in I :\ f_i(x^*) - v^* g_i(x^*) - q^* = 0 \}$ and $J(x^*) = \{ j \in J : h_j(x) = 0 \}$.

Then x* is (FP) – optimal with the corresponding optimal objective value equal to v*.

Proof: Let x* be not an optimal solution of (FP). Then it follows from lemma 2.4.3 that (x*, v*, q*) is not optimal for (EP_v) with

$$q < q^*, \ (x \neq x^*).$$

Using (4) and (14) , the inequality implies

$$f_i(x) - vg_i(x) \le q < q^* = f_i(x^*) - v^* g_i(x^*), i \in I(x^*)$$

$$f_i(x) - vg_i(x) < f_i(x^*) - v^* g_i(x^*).$$

Also, from (12) it follows that $\lambda_i^* = 0$ for each $i \notin I(x^*)$ and therefore

$\sum_{i=1}^{k} \lambda_i^* = 1$, which ensures the existence of at least one $\lambda_i^* > 0$, $i \in I(x^*)$. Hence by multiplying each of the above inequalities

by λ_i^* and $\theta_i(x,x^*) > 0$, $i \in I(x^*)$ and adding , we obtain

$$\sum_{i \in I(x^*)} \theta_i(x, x^*) \lambda_i^* \left(f_i(x) - vg_i(x) \right) < \sum_{i \in I(x^*)} \theta_i(x, x^*) \lambda_i^* \left(f_i(x^*) - v^* g_i(x^*) \right).$$

Now, vector-ρ - η -pseudo convex of $f_i(x) - vg_i(x)$, $i \in I(x^*)$ implies

$$\sum_{i\in I(x^*)} \lambda_i \xi_i \eta^T(x, x^*) + \rho\|\varphi(x,u)\|^2 < 0,$$

$$\forall \xi_i \in \partial\{ f_i(x^*) - v^* g_i(x^*) \},\ i \in I(x^*). \qquad (19)$$

Again, using $\lambda_i^* = 0$, $i \notin I(x^*)$, we can rewrite (19) as

$$\sum_{i\in I(x^*)} \lambda_i \xi_i \eta^T(x, x^*) + \rho\|\varphi(x,u)\|^2 < 0,$$

$$\forall \xi_i \in \partial\{ f_i(x^*) - v^* g_i(x^*) \},\ i = 1,2,..,k. \qquad (20)$$

Now, from (5),(13)and (18), it follows

$\mu_j h_j(x) \le \mu_j^* h_j(x^*)$, $j \in J(x^*)$. Again, multiply each of the above inequality by $\theta_j(x,x^*) > 0; j \in J(x^*)$ and

adding,we get $\sum_{j\in J(x^*)} \theta_j(x, x^*)\mu_j^* h_j(x) \le \sum_{j\in J(x^*)} \theta_j(x, x^*)\mu_j^* h_j(x^*)$.

Now, vector-ρ - η - quasi convexity of $h_j(x)$, $j \in J(x^*)$ implies

$$\sum_{j\in J(x^*)} \mu_j^* \alpha_j \eta^T(x, x^*) \le -\ \rho^1\|\varphi(x, x^*)\|^2,\ \forall \alpha_j \in \partial h_j(x^*),\quad j \in J(x^*).$$

Since, $\mu_j^* = 0$, $j \notin J(x^*)$, therefore, without loss of generality, we can write above inequality as

$$\sum_{j\in J(x^*)} \mu_j^* \alpha_j \eta^T(x, x^*) \le -\ \rho^1\|\varphi(x, x^*)\|^2,\ \forall \alpha_j \in \partial h_j(x^*),\ j = 1,2,\ldots,m. \ (21)$$

Adding (20) and (21), we get

$$\{\sum_{i=1}^{k}\lambda_i^*\xi_i + \sum_{j=1}^{m}\mu_j^*\alpha_j\}\eta^T(x, x^*) < -(\rho + \rho^1)\|\varphi(x, x^*)\|^2,$$

$$\forall\xi_i \in \partial\{f_i(x^*) - v^* g_i(x^*)\}, i = 1,2,..,k, \ \forall\alpha_j \in \partial h_j(x^*), \ j = 1,2,..,m.$$

$$\{\sum_{i=1}^{k}\lambda_i^*\xi_i + \sum_{j=1}^{m}\mu_j^*\alpha_j\}\eta^T(x, x^*) < 0 \text{ (using assumption c).}$$

which is a contradiction to (11).

Hence x* is an optimal solution of (FP), and consequently, (x*.,v*,q*) is an optimal solution for (EP_{v^*}) with optimal value q* = 0 = min q.

Therefore, by lemma 4.3, x* is (FP) – optimal with v* as the corresponding optimal value of the (FP)-objective.

6. WEAK AND STRONG DUALITY RESULTS:

6.1. Weak Duality Theorem:

Let (x,v,q) $\in$T and (u,y, λ,μ) $\in$ W be arbitrary feasible solutions of (EP_v) and (FD) respectively. Further, assume that

(a) $(f_i - vg_i), i \in I$ be vector-ρ - η - pseudoconvex with respect to η.

(b) h_j , $j \in J(x^*)$ vector-ρ - η - quasi convex with respect to η and

(c) $(\rho + \rho^1) > 0$, where, I* = $\{i \in I : \lambda > 0\}$ and J*= $\{ j \in J : \mu_j >= 0\}$.

Then, $q \geq y$.

Proof: Suppose, contrary to the result of the theorem

$$q < y.$$

Using (4), it implies that $f_i(x) - vg_i(x) \le q \le y,\ i \in I.$

For $i \in I^*$, it follows from (2.7), in view of the above inequality that

$$f_i(x) - vg_i(x) < f_i(u) - vg_i(u)\ i \in I^*.$$

Hence, by multiplying each of the above inequality by λ_i and

$\theta_j(x,u) > 0; i \in I^*$, and adding, we get

$$\sum_{i \in I^*} \theta_i(x,u)\lambda_i \{f_i(x) - vg_i(x)\} < \sum_{i \in I^*} \theta_i(x,u)\lambda_i \{f_i(u) - vg_i(u)\}.$$

Now, vector-ρ - η - pseudoconvex of $(f_i - vg_i), i \in I^*$implies

$$\sum_{i \in I^*} \lambda_i \xi^1 \eta^T(x,u) < -\rho^1 \|\varphi(x,x^*)\|^2, \quad \forall \xi_i^1 \in \partial\{f_i(u) - vg_i(u)\},\ i \in I^*. \qquad (22)$$

Again, using $\lambda_i = 0, i \notin I^*$, we can rewrite (22) as

$$\sum_{i=1}^{k} \lambda_i \xi^1 \eta^T(x,u) < -\rho^1 \|\varphi(x,x^*)\|^2, \quad \forall \xi_i^1 \in \partial\{f_i(u) - vg_i(u)\},\ i = 1,2,..,k. \qquad (23)$$

Also, for $j \in J(x^*)$, (5) and (8) yield

$$\mu_j h_j(x) \le \mu_j^* h_j(x^*),\ j \in J^*.$$

Again, multiply each of the above inequality by $\theta_j(x,u) > 0; j \in J^*$,

we get $$\sum_{j\in J^*} \theta_j(x,u)\mu_j h_j(x) \leq \sum_{j\in J^*} \theta_j(x,u)\mu_j h_j(u).$$

Now, vector-ρ - η - - quasi convexity of h_j, $j \in J^*$ implies

$$\sum_{j\in J^*} \mu_j \alpha_j^1 \eta^T(x,u) \leq - \rho^1 \|\varphi(x,u)\|^2, \quad \forall \alpha_j^1 \in \partial h_j(u), \quad j \in J^*. \tag{24}$$

Since, $\mu_j = 0$, $j \notin J^*$, without loss of generality, we can rewrite above inequality as

$$\sum_{j\in J^*} \mu_j \alpha_j^1 \eta^T(x,u) \leq - \rho^1 \|\varphi(x,u)\|^2, \quad \forall \alpha_j^1 \in \partial h_j(u), \quad j = 1,2,\ldots,m. \tag{25}$$

Adding (23) and (24), we get

$$\{\sum_{i=1}^{k} \lambda_i \xi_i^1 + \sum_{j=1}^{m} \mu_j \alpha_j^1\}\eta^T(x,u) < -(\rho + \rho^1)\|\varphi(x,u)\|^2,$$

$$\forall \xi_i^1 \in \partial\{f_i(u) - vg_i(u)\}, \; i = 1,2,..,k, \; \forall \alpha_j^1 \in \partial h_j(u), \; j = 1,2,..,m.$$

$$\{\sum_{i=1}^{k} \lambda_i \xi_i^1 + \sum_{j=1}^{m} \mu_j \alpha_j^1\}\eta^T(x,u) < 0 \text{ (using assumption c).}$$

Which is contradiction to (7).

6.2. Strong Duality theorem:

Let $(\bar{x}, \bar{v}, \bar{q}) \in T$ be (EP_v)- optimal at which an appropriate constraint qualification holds . Then there exist $\bar{\lambda}$ and $\bar{\mu}$ such that $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{\mu})$ is (FD)-feasible and the corresponding objective values for (EP_v) and (FD) are equal. If also, the hypothesis of theorem.6.1. holds, $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{\mu})$ is (FD)-optimal.

Proof: Using necessary theorem, there exists

$\bar{\lambda} \in R^k, \bar{\mu} \in R^m$ such that 11 to18 holds.

Where $V^* = \dfrac{f_i(x^*)}{g_i(x^*)}$, i= 1,2,…..,k,

there fore $(\bar{x}, \bar{y}, \bar{\lambda}, \bar{\mu})$ is feasible for (FD) suppose $(\bar{x}, \bar{v}, \bar{\lambda}, \bar{\mu})$ is not efficient solution of (FD), then there exists a feasible

solution (u,v,λ, μ) of (FD) such that

$$\bar{\lambda}_i \{ f_i(\bar{x}) - \bar{v} g_i(\bar{x}) \} \leq \bar{\lambda}_i \{ f_i(\bar{u}) - \bar{v} g_i(\bar{u}) \}$$

and

$$\bar{\lambda}_j \{ f_j(\bar{x}) - \bar{v} g_j(\bar{x}) \} < \bar{\lambda}_j \{ f_j(\bar{u}) - \bar{v} g_j(\bar{u}) \}, \text{ for some j.}$$

Which contradicts to weak duality theorem as

$$\bar{\mu}_j \, h_j(\bar{x}) = 0.$$

Hence the proof.

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PERSONAL INTRODUCTION

Mrs.G.Sreelatha completed her Msc [mathematics] from Osmania university in Hyderabad in Telangana state, India. Since then she has been teaching for degree and engineering students for the past 20 years. Formerly she served at Jawahar Bharathi degree collage, kavali, Nellore [dt] for 13 years. At present she is serving at DBS institute of technology, maddurupadu, Nellore[dt] since 2009. She has attended and presented papers in four international conferences. Firstly she has given a paper presentation at Madurai in Thaigarajar collage of engineering in the international conference. Later she has given paper presentations simultaneously in Nagpur [ICMS], Cochin [IMRF] and Simla [Chitkara] international conferences. She has submitted for Phd at dravida university, kuppam in Andhra Pradesh in the research area of Some of the classical optimisation techniques used in multi objective fractional programming problem involving partial vector convexity under the guidance of Dr.G.Varalakshmi [Msc] [Phd], professor in DKDC women's college.

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