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SECOND QUANTIZATION OF ELECTROMAGNETIC FIELD IN TERMS OF COMPLEX ISOTROPIC VECTORS

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ABSTRACT

In previous works, Maxwell's equations for electromagnetic field have been written in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$. It has been proved, that in the case of electromagnetic field in vacuum, the solution of Maxwell's equations satisfies non-linear condition $\vec{F}^2 = 0$, i.e., the complex vector $\vec{F} = \vec{E} + i\vec{H}$ is isotropic vector. The last condition is equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by equating to zero separately real and imaginary parts of equality $\vec{F}^2 = 0$. Furthermore, the Lagrange formalism for electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated.

In this work, in development of this formalism, we elaborated the second quantization of electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$.

***Keywords:* Electromagnetic field, complex isotropic vector, second quantization.**

INTRODUCTION

Classical physics is dominated by two fundamental concepts. The first is the concept of a particle, a discrete entity with definite position and momentum which moves in accordance with Newton's laws of motion. The second is the concept of an electromagnetic wave, an extended physical entity with a presence at every point in space that is provided by electric and magnetic fields, which change in accordance with Maxwell's laws of electromagnetism expressed by Maxwell's equations which give a complete description of classical electric and magnetic phenomena.

The wave equations were able to describe the quantum mechanical behavior of a single particle in covariant manner. Such a treatment is referred to as first quantization. It is suitable for description of the interactions of massive particles with kinetic energies much less than the particle rest mass, where energy conservation forbids the production of real particle-antiparticles pairs. However, at higher energies where the production of single particles or particle-antiparticles pairs is energetically possible, the first quantization form fails completely, reason why we introduce a new quantization scheme capable of describing particle production and annihilation fully. Such a quantization scheme is referred to as second quantization.

In previous works, Maxwell's equations for electromagnetic field have been written in terms of complex vectors $\vec{F} = \vec{E} + i\vec{H}$. It has been proved, that the solution of Maxwell's equations for vacuum (with zero at right side) satisfies non-linear condition $\vec{F}^2 = 0$, equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by separating real and imaginary parts in equality $\vec{F}^2 = 0$. In the works that followed, the Lagrange formalism for electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated.

In this work, in development of this formalism, we shall elaborate the second quantization of electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$.

RESEARCH METHOD

In previous works, Maxwell's equations for electromagnetic field have been written in terms of complex vectors $\vec{F} = \vec{E} + i\vec{H}$. Their solution has been obtained in the form of plane waves. Furthermore, the Lagrange formalism of electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated. Expressions for dynamical variables (energy, momentum, charge and spin) conserved in time have been obtained in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$. Using Fourier transformation, these dynamical variables will be written as Eigen-values of operators, satisfying the corresponding commutation relations.

Maxwell's Equations in Terms of Isotropic Complex Vectors and Their Solution

Let us introduce complex vector

$$\vec{F} = \vec{E} + i\vec{H} \quad , \qquad (1)$$

where $\vec{E}, \vec{H}$ are electric and magnetic field strengths. Then, Maxwell's equations for vacuum

$$\begin{cases} \vec{\nabla} \times \vec{E} + \frac{\partial \vec{H}}{\partial t} = 0 \\ \vec{\nabla} \times \vec{H} - \frac{\partial \vec{E}}{\partial t} = 0 \\ \vec{\nabla}.\vec{E} = 0 \\ \vec{\nabla}.\vec{H} = 0 \end{cases} \qquad (2)$$

can be written in the form

$$\begin{cases} D^0 \vec{F} = i\vec{D} \times \vec{F} \\ \vec{D}\vec{F} = 0 \end{cases} \qquad (3)$$

Here

$$D^0 = \frac{i}{2}\frac{\partial}{\partial t} \qquad (4)$$

$$\vec{D} = -\frac{i}{2}\vec{\nabla} \qquad (5)$$

The solution of equations (3) can be written in the form

$$\vec{F} = \vec{F}^0 e^{-2ikt + 2i\vec{k}\vec{r}}, \qquad (6)$$

Where

$$\vec{F}^0 = \frac{i}{2}\begin{bmatrix} \sin\varphi + is\cos\vartheta\cos\varphi \\ -\cos\varphi + is\cos\vartheta\sin\varphi \\ -is\sin\vartheta \end{bmatrix}, \qquad (7)$$

$k = |\vec{k}|$ is the energy, $s = \pm 1$ is the polarization and φ, ϑ are the angles of orientation of the wave vector $\vec{k}$, chosen so that $k_1 + ik_2 = k\sin\vartheta\, e^{i\varphi}$, $k_3 = k\cos\vartheta$.

Separating real and imaginary parts in formula (7), we obtain the solution for field strengths

$$\vec{E} = \frac{1}{2}\begin{bmatrix} \sin\varphi \\ -\cos\varphi \\ 0 \end{bmatrix} e^{-2ikt + 2i\vec{k}\vec{r}} \qquad (8)$$

$$\vec{H} = \frac{1}{2}\begin{bmatrix} s\cos\vartheta\cos\varphi \\ s\cos\vartheta\sin\varphi \\ -s\sin\vartheta \end{bmatrix} e^{-2ikt + 2i\vec{k}\vec{r}} \quad . \qquad (9)$$

Second Quantization of Electromagnetic Field in Terms of Complex Isotropic Vectors

In previous work, it has been proved, that Maxwell's equations (3) can be obtained from the Lagrange function

$$L = \frac{i}{2}\{[D^0\vec{F} - i\vec{D}\times\vec{F}]\vec{F}^* - [D^0\vec{F}^* + i\vec{D}\times\vec{F}^*]\vec{F}\}/\left(\frac{\vec{F}\vec{F}^*}{2}\right)^{1/2}. \quad (10)$$

Using Noether's theorem, from the Lagrange function (10), we find the following expressions for dynamical variables (energy, momentum, charge and spin) conserved in time.

For energy we find

$$E = \int T^{00} d^3x, \quad (11)$$

Where

$$T^{00} = \frac{i}{2}\left(\vec{F}^* \frac{\partial\vec{F}}{\partial t}\right)/\left(\frac{\vec{F}\vec{F}^*}{2}\right)^{1/2} = k|\vec{E}|. \quad (12)$$

For momentum, we obtain

$$P^j = \int T^{0j} d^3x, \quad (13)$$

Where

$$T^{0j} = \frac{i}{2}\left(\vec{F}^* \frac{\partial\vec{F}}{\partial x_j}\right)/\left(\frac{\vec{F}\vec{F}^*}{2}\right)^{1/2} = k_j|\vec{E}|. \quad (14)$$

For charge, we have

$$Q = \int J^0 d^3x\,, \quad (15)$$

where

$$J^0 = \frac{i}{2}(\vec{F}^*\vec{F})/\left(\frac{\vec{F}\vec{F}^*}{2}\right)^{1/2} = |\vec{E}|. \quad (16)$$

Finally, for spin pseudo vector, we find

$$\vec{S} = \frac{i}{2}(\vec{F}^* \times \vec{F})/\left(\frac{\vec{F}\vec{F}^*}{2}\right)^{1/2} = \frac{\vec{E}\times\vec{H}}{|\vec{E}|}. \quad (17)$$

Let us expand the wave function $\vec{F}(x)$ in Fourier series

$$\vec{F}(x) = \sum_{k,s} a_s(\vec{k})\vec{F}_s(\vec{k})e^{-2ikt+2i\vec{k}\vec{r}}, \qquad (18)$$

$$\vec{F}^*(x) = \sum_{k,s} a_s^*(\vec{k})\vec{F}_s^*(\vec{k})e^{2ikt-2i\vec{k}\vec{r}}. \qquad (19)$$

Replacing formulas (18) and (19) in formulas (12), (14). (16) and (17) and considering the normalization condition

$$\int \left[\frac{\vec{F}_{ks}^* \vec{F}_{k's'}}{2(\vec{F}_{ks}^* \vec{F}_{k's'}/2)^{1/2}}\right] d^3x = \delta_{kk'}\delta_{ss'}, \qquad (20)$$

we obtain

$$E = \sum_{k,s} k[a_s^*(\vec{k})a_s(\vec{k})], \qquad (21)$$

$$P_j = \sum_{k,s} k_j[a_s^*(\vec{k})a_s(\vec{k})], \qquad (22)$$

$$Q = \sum_{k,s}[a_s^*(\vec{k})a_s(\vec{k})], \qquad (23)$$

$$S_j = \sum_{k,s} æ_j[a_s^*(\vec{k})a_s(\vec{k})]. \qquad (24)$$

Exchanging $a_s(\vec{k})$ by operator $\hat{a}_s(\vec{k})$, we find

$$\hat{E} = \sum_{k,s} k[\hat{a}_s^+(\vec{k})\hat{a}_s(\vec{k})], \qquad (25)$$

$$\hat{P}_j = \sum_{k,s} k_j[\hat{a}_s^+(\vec{k})\hat{a}_s(\vec{k})], \qquad (26)$$

$$\hat{Q} = \sum_{k,s}[\hat{a}_s^+(\vec{k})\hat{a}_s(\vec{k})], \qquad (27)$$

$$\hat{S}_j = \sum_{k,s} æ_j[\hat{a}_s^+(\vec{k})\hat{a}_s(\vec{k})]. \qquad (28)$$

Here $\vec{æ}$ is a unit vector in the direction of the polarization vector $\vec{S}$.

To ensure the positive determination of energy, we must require the following commutation relations

$$[\hat{a}_s^+(\vec{k}), \hat{a}_{s'}(\vec{k}')]_- = \delta_{kk'}\delta_{ss'}. \qquad (29)$$

Using formulas (25)-(28), we find expressions for Eigen-values of the above operators

$$E = \sum_{k,s} k[N_{ks}], \qquad (30)$$

$$P_j = \sum_{k,s} k_j [N_{ks}], \tag{31}$$

$$Q = \sum_{k,s} [N_{ks}], \tag{32}$$

$$S_j = \sum_{k,s} æ_j [N_{ks}]. \tag{33}$$

Here N_{ks} is the number of particles.

DISCUSSION AND CONCLUSION

In previous works, Maxwell's equations for electromagnetic field have been written through complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$ and the Lagrange formalism for electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated. In this work, we developed the second quantization of electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$. Here, dynamical variables (energy, momentum, charge and spin) conserved in time have been expressed through the number of particles.

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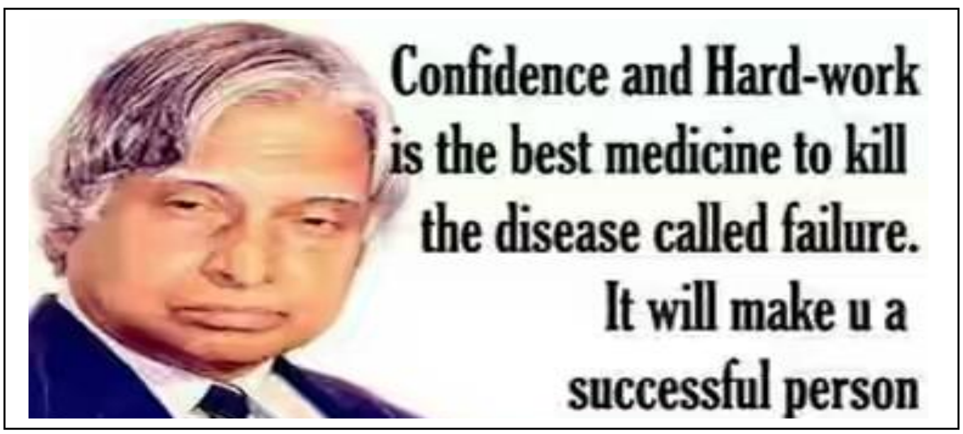
Confidence and Hard-work
is the best medicine to kill
the disease called failure.
It will make u a
successful person