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SECOND QUANTIZATION FOR NEUTRINO FIELD IN TERMS OF COMPLEX ISOTROPIC VECTORS

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ABSTRACT

In previous works, using Cartan map, spinor Weyl's equation for neutrino has been written in tensor form, in the form of non-linear Maxwell's like equations through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. It has been proved, that the complex vector $\vec{F} = \vec{E} + i\vec{H}$ satisfies non-linear condition $\vec{F}^2 = 0$. The last condition is equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by equating to zero separately real and imaginary parts of equality $\vec{F}^2 = 0$. Furthermore, the Lagrange formalism for neutrino field in terms of isotropic complex vectors $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated.

In this work, in development of the above formalism, we elaborated the second quantization for neutrino field in terms of isotropic complex vectors $\vec{F} = \vec{E} + i\vec{H}$.

***Keywords:* Neutrino field, second quantization, isotropic complex vector.**

INTRODUCTION

In previous works, using Cartan map, spinor Weyl's equation for neutrino has been written in tensor form, in the form of non-linear Maxwell's like equations through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. It has been proved, that the complex vector $\vec{F} = \vec{E} + i\vec{H}$ satisfies non-linear condition $\vec{F}^2 = 0$. The last condition is equivalent to two conditions for real quantities $\vec{E}^2 - \vec{H}^2 = 0$ and $\vec{E}.\vec{H} = 0$, obtained by equating to zero separately real and imaginary parts of equality $\vec{F}^2 = 0$. The vectors $\vec{E}$ and $\vec{H}$ have the same properties as those of vectors $\vec{E}$ and $\vec{H}$, components of electromagnetic field. For example, under Lorentz relativistic transformations, they are transformed as vectors $\vec{E}$ and $\vec{H}$, components of the electromagnetic tensor $F_{\mu\nu}$. In addition, it has been proved, that the solution of these non-linear equations for free particle as well fulfills Maxwell's equations for vacuum (with zero at right side). This enables us to interpret neutrino field as another form of electromagnetic field but with half spin. Furthermore, the Lagrange formalism for neutrino field in terms of isotropic complex vectors $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated.

In this work, we shall develop the second quantization for neutrino field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$. Here, dynamical variables (energy, momentum, charge and spin) conserved in time, will be expressed through the number of particles and the number of antiparticles.

RESEARCH METHOD

In previous works, using Cartan map, spinor Weyl's equation for neutrino has been written in tensor form, in the form of non-linear Maxwell's like equations through isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$. Using the same method, the Lagrange formalism for neutrino field in terms of isotropic complex vector $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated.

In this work, using the traditional procedure of second quantization, we shall develop the second quantization for neutrino field in terms of isotropic complex vectors $\vec{F} = \vec{E} + i\vec{H}$.

Second Quantization for Neutrino Field in Terms of isotropic Complex Vectors

Weyl's equation for neutrino has the form

$$p_0\xi = (\vec{p}.\vec{\sigma})\xi \,. \tag{1}$$

With the help of Cartan map, spinor Weyl's equation (1) has been written through isotropic complex vector $\vec{F}$=$\vec{E}$+i$\vec{H}$, satisfying non-linear condition $\vec{F}^2$ =0, as follows

$$D^0\vec{F} = i\,\vec{D} \times \vec{F} - \left(\vec{D}F_i\right)v_i \,, \tag{2}$$

Where

$$D^0 = \frac{i}{2}\frac{\partial}{\partial t} \,, \tag{3}$$

$$\vec{D} = -\frac{i}{2}\vec{\nabla} \,, \tag{4}$$

$$\vec{v} = \frac{\vec{J}}{J_0} = \frac{\vec{E}\times\vec{H}}{\left|\vec{E}\right|^2} \,. \tag{5}$$

Here and in the following we shall use the natural system of units in which c=ħ=1.

Equation (2), written through $\vec{E}$ and $\vec{H}$, can be represented in the form of a system of non-linear Maxwell's like equations

$$\begin{cases} \mathrm{rot}\vec{E} + \frac{\partial \vec{H}}{\partial t} = v_i(\vec{\nabla} H_i) \\ \mathrm{rot}\vec{H} - \frac{\partial \vec{E}}{\partial t} = -v_i(\vec{\nabla} E_i) \end{cases} . \qquad (6)$$

In the notations of complex isotropic vectors, Maxwell's equations for vacuum take the form

$$\begin{cases} D^0\vec{F} = i\,\vec{D} \times \vec{F} \\ \vec{D}\vec{F} = 0 \end{cases} . \qquad (7)$$

However, in the general case, the solution of Maxwell's equations does not satisfy isotropic condition $\vec{F}^2 = 0$, whereas the solution of Weyl's equation (2) always satisfies this condition.

In spinor formalism, Weyl's equation (1) can be derived by variation principle from Lagrange function

$$L = \frac{i}{2}\left(\xi\sigma^\mu \partial_\mu \xi^* - \partial^\mu \xi \sigma_\mu \xi^*\right) . \qquad (8)$$

Transforming formula (8) according to Cartan map, we obtain

$$L = \frac{1}{2}\{[D_0\vec{F} - i\vec{D} \times \vec{F} + v_i(\vec{D}F_i)]\vec{F}^* - [D_0\vec{F}^* + i\vec{D} \times \vec{F}^* + v_i(\vec{D}F_i^*)]\vec{F}\}/(\vec{F}\vec{F}^*/2)^{1/2}. \quad (9)$$

Formula (9), written through components of vectors $\vec{F}$ and $\vec{F}^*$takes the form

$$L = \frac{i}{4}\left\{\left[\frac{\partial F_i}{\partial t} + i\varepsilon_{ijk}\frac{\partial F_k}{\partial x_j} - v_j\left(\frac{\partial F_j}{\partial x_i}\right)\right] F_i^* - \left[\frac{\partial F_i^*}{\partial t} - i\varepsilon_{ijk}\frac{\partial F_k^*}{\partial x_j} - v_j\left(\frac{\partial F_j^*}{\partial x_i}\right)\right] F_i\right\}/(F_iF_i^*/2)^{1/2}. \quad (10)$$

In calculating variations in formula (10), expression $(\vec{F}\vec{F}^*/2)^{1/2}$ will be considered as a constant.

Using Noether's theorem, we can derive from the Lagrange function (9), expressions for fundamental physical dynamical variables, conserved in time.

Energy is determined by the formula

$$E = \int T^{00} d^3x \,, \tag{11}$$

where

$$T^{00} = \frac{i}{4}\left[\vec{F}^* \frac{\partial \vec{F}}{\partial t} - \vec{F}\frac{\partial \vec{F}^*}{\partial t}\right] / \left(\vec{F}\vec{F}^*/2\right)^{1/2} . \tag{12}$$

With consideration of expression

$$\vec{F} = \left(\vec{E}^0 + i\vec{H}^0\right) e^{-2i\varepsilon kt + 2i\vec{k}\vec{r}} \,, \tag{13}$$

we find

$$T^{00} = \varepsilon k \left|\vec{E}\right| . \tag{14}$$

Similarly, for momentum we have

$$P^j = \int T^{0j} d^3x \,, \tag{15}$$

where

$$T^{0j} = \frac{i}{4}\left[\left(\vec{F}\vec{\nabla}_j\vec{F}^*\right) - \left(\vec{F}^*\vec{\nabla}_j\vec{F}\right)\right] / \left(\vec{F}\vec{F}^*/2\right)^{1/2} . \tag{16}$$

With consideration of expression (13), we obtain

$$\vec{P} = \vec{k}\left|\vec{E}\right| . \tag{17}$$

For charge, we have

$$Q = \int j^0 d^3x \,, \tag{18}$$

Where

$$j^0 = \frac{1}{4}\left(\vec{F}\vec{F}^* + \vec{F}^*\vec{F}\right) / \left(\vec{F}\vec{F}^*/2\right)^{1/2} . \tag{19}$$

Replacing formula (13) into formula (19), we find

$$j^0 = |\vec{E}| . \tag{20}$$

The density of the spin pseudo vector is determined by the formula

$$S_k = \frac{1}{2} \varepsilon_{ijk} \; S_{lm}, \tag{21}$$

Where

$$S^0_{lm} = - \frac{\partial L}{\partial F_{i,0}} F_j A^j_{i,lm} - \frac{\partial L}{\partial F^*_{i,0}} F^*_j A^j_{i,lm} \; . \tag{22}$$

Here

$$A^j_{i,lm} = g_{il} \delta^j_m - g_{im} \delta^j_l \; . \tag{23}$$

Replacing formula (9) and formula (23) in formula (22), we find

$$S^0_{lm} = \frac{i}{4} (F_l F^*_m - F_m F^*_l) / (\vec{F}\vec{F}^*/2)^{1/2} \; . \tag{24}$$

Thus, from formula (21) and formula (24) we have

$$\vec{S} = i\,(\vec{F} \times \vec{F}^*) / (\vec{F}\vec{F}^*/2)^{1/2} \; . \tag{25}$$

Using formula (13), expression for spin pseudo vector (25) can be rewritten through vectors $\vec{E}$ and $\vec{H}$ as follows

$$\vec{S} = \frac{\vec{E} \times \vec{H}}{|\vec{H}|} \; . \tag{26}$$

In previous work, we proved that the non-linear equation (2) admits the solution with positive and negative energies $\vec{F}^+(x)$ and $\vec{F}^-(x)$.

Let

$$\vec{F}^+(x) = \vec{F}^{0+}(\vec{k})\; e^{-2ikt + 2i\vec{k}\vec{r}} \; , \tag{27}$$

$$\vec{F}^-(x) = \vec{F}^{0-}(\vec{k})\; e^{2ikt + 2i\vec{k}\vec{r}} \; . \tag{28}$$

Then, the general solution is

$$\vec{F}(x) = a\,\vec{F}^{0+}(\vec{k})\,e^{-2ikt+2i\vec{k}\vec{r}} + b\,\vec{F}^{0-}(\vec{k})\,e^{2ikt+2i\vec{k}\vec{r}}\,, \qquad (29)$$

Or

$$\vec{F}(x) = \left[a\,\vec{F}^{0+}(\vec{k})\,e^{-2ikt} + b\,\vec{F}^{0-}(\vec{k})\,e^{2ikt}\right] e^{2i\vec{k}\vec{r}}\,. \qquad (30)$$

Let us expand the function $\vec{F}(x)$ in Fourier series

$$\vec{F}^{+}(x) = \sum_k a^{+}(\vec{k})\,\vec{F}^{+}(\vec{k})\,e^{-2ikt+2i\vec{k}\vec{r}}\,, \qquad (31)$$

$$\vec{F}^{-}(x) = \sum_k a^{-}(\vec{k})\,\vec{F}^{-}(\vec{k})\,e^{2ikt+2i\vec{k}\vec{r}}\,. \qquad (32)$$

Exchanging in the last formula $\vec{k} \to -\vec{k}$ and adopting new notations $a^{+}(\vec{k}) = a(\vec{k})$, $a^{-}(-\vec{k}) = b^{*}(\vec{k})$, we obtain

$$\vec{F}(x) = \vec{F}^{+}(x) + \vec{F}^{-}(x) = \sum_k a(\vec{k})\,\vec{F}^{+}(\vec{k})\,e^{-2ikt+2i\vec{k}\vec{r}} + b^{*}(\vec{k})\vec{F}^{-}(-\vec{k})\,e^{2ikt-2i\vec{k}\vec{r}}\,, \qquad (33)$$

$$\vec{F}^{*}(x) = \vec{F}^{+*}(x) + \vec{F}^{-*}(x) = \sum_k a^{*}(\vec{k})\,\vec{F}^{+*}(\vec{k})\,e^{2ikt-2i\vec{k}\vec{r}} + b(\vec{k})\vec{F}^{-*}(-\vec{k})\,e^{-2ikt+2i\vec{k}\vec{r}}. \qquad (34)$$

Replacing (33)-(34) in formulas (11), (15), (18), (25) and considering the normalization conditions

$$\int \frac{\vec{F}_k^{\pm *}(x)\vec{F}_{k'}^{\pm}(x)}{2\left(\vec{F}_k^{\pm *}(x)\vec{F}_{k'}^{\pm}(x)/2\right)^{1/2}}\,d^3x = \delta_{kk'}\,, \qquad (35)$$

we obtain the following expressions for physical observable quantities

$$E = \sum_k k\left[a^{*}(\vec{k})a(\vec{k}) - b(\vec{k})b^{*}(\vec{k})\right], \qquad (36)$$

$$P_j = \sum_k k_j\left[a^{*}(\vec{k})a(\vec{k}) - b(\vec{k})b^{*}(\vec{k})\right], \qquad (37)$$

$$Q = \sum_k\left[a^{*}(\vec{k})a(\vec{k}) + b(\vec{k})b^{*}(\vec{k})\right], \qquad (38)$$

$$S_j = \sum_k æ_j(\vec{k})\left[a^{*}(\vec{k})a(\vec{k}) + b(\vec{k})b^{*}(\vec{k})\right]. \qquad (39)$$

Here $\vec{æ} = \frac{\vec{E}\times\vec{H}}{\vec{E}^2}$ is the unit vector in the direction of the spin vector.

Exchanging the quantities $a(\vec{k})$, $b(\vec{k})$ into operators $\hat{a}(\vec{k})$ and $\hat{b}(\vec{k})$, we find

$$\hat{\vec{F}}(x) = \; = \sum_k \hat{a}(\vec{k})\, \vec{F}^{+}(\vec{k})\; e^{-2ikt+2i\vec{k}\vec{r}} + \hat{b}^{+}(\vec{k})\vec{F}^{-}(-\vec{k})\; e^{2ikt-2i\vec{k}\vec{r}}, \tag{40}$$

$$\hat{E} = \sum_k k\left[\hat{a}^{+}(\vec{k})\hat{a}(\vec{k}) - \hat{b}(\vec{k})\hat{b}^{+}(\vec{k})\right], \tag{41}$$

$$\hat{P}_j = \sum_k k_j\left[\hat{a}^{+}(\vec{k})\hat{a}(\vec{k}) - \hat{b}(\vec{k})\hat{b}^{+}(\vec{k})\right], \tag{42}$$

$$\hat{Q} = \sum_k\left[\hat{a}^{+}(\vec{k})\hat{a}(\vec{k}) + \hat{b}(\vec{k})\hat{b}^{+}(\vec{k})\right], \tag{43}$$

$$\hat{S}_j = \sum_k æ_j\left[\hat{a}^{+}(\vec{k})\hat{a}(\vec{k}) + \hat{b}(\vec{k})\hat{b}^{+}(\vec{k})\right]. \tag{44}$$

To ensure the positive determination of energy, we must require the following anti-commutation relations

$$\left[\hat{a}^{+}(\vec{k}), \hat{a}(\vec{k}')\right]_{+} = \left[\hat{b}^{+}(\vec{k}), \hat{b}(\vec{k}')\right]_{+} = \delta_{kk'}, \tag{45}$$

the other anti-commutators are equal to zero.

As result, for Eigen-values of operators for physical quantities, we obtain

$$E = \sum_k k[N_k + \bar{N}_k], \tag{46}$$

$$P_j = \sum_k k_j[N_k + \bar{N}_k], \tag{47}$$

$$Q = \sum_k[N_k - \bar{N}_k], \tag{48}$$

$$S_j = \sum_k æ_j[N_k - \bar{N}_k]. \tag{49}$$

Here N_k is the number of particles and $\bar{N}_k$ is the number of anti-particles.

DISCUSSION AND CONCLUSION

In previous works, via Cartan map, spinor Weyl's equation for neutrino has been written in tensor form, in the form of non-linear Maxwell's like equations through complex isotropic vector $\vec{F} = \vec{E} + i\vec{H}$. Furthermore, the Lagrange formalism for neutrino field in tensor formalism, in terms of complex isotopic vectors $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated. In this work, we elaborated the second quantization for neutrino field in terms of isotropic complex vectors $\vec{F} = \vec{E} + i\vec{H}$. We found expressions for dynamical variables (energy, momentum, charge and spin) conserved in time, through the number of particles and the number of antiparticles.

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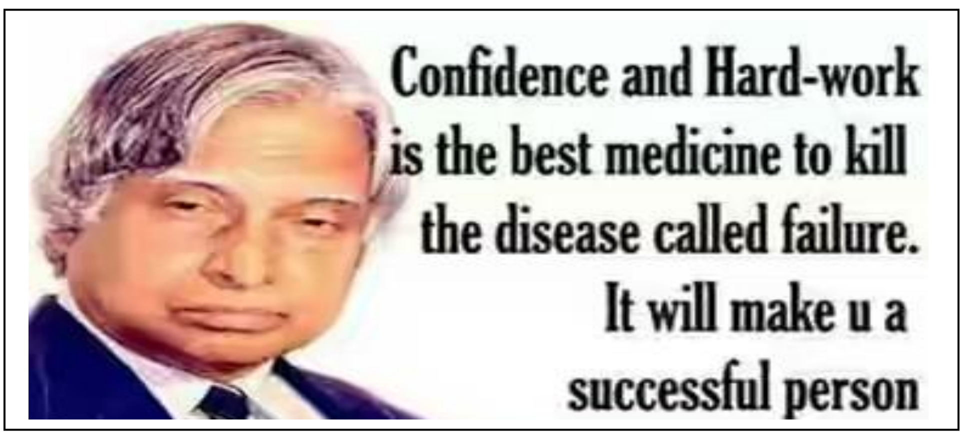
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