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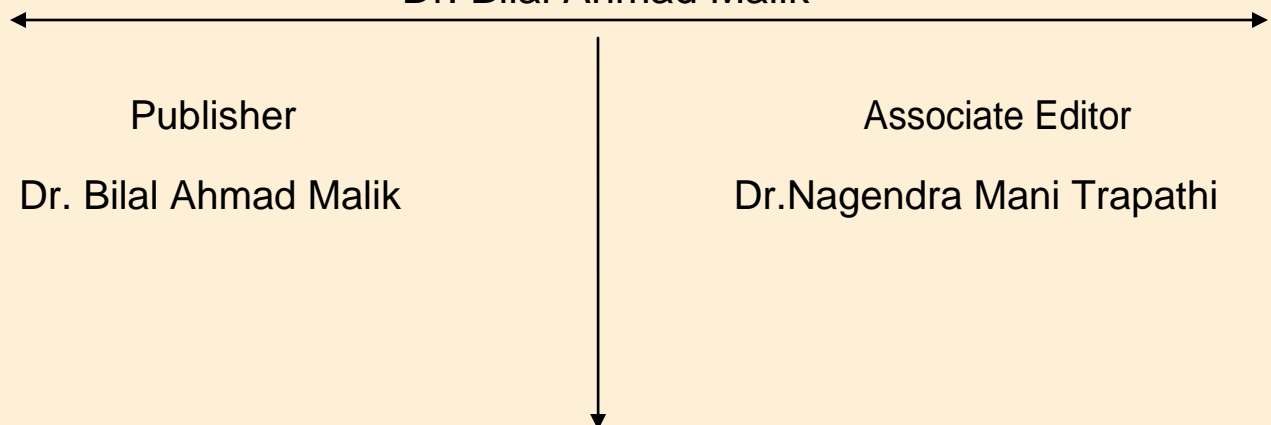
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## CALCULATION OF ELECTRICAL RESISTIVITY OF LIQUID METALS USING THE PERCUS YEVICK STRUCTURE FACTOR AND MODEL POTENTIAL

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### ABSTRACT

*In this work, the concept of charge density was employed. The charge density incorporated in the Model potential with the Percus Yevick structure factor was used to calculate electrical resistivity of liquid metals. The results obtained are in perfect agreement with the experimentally determined values. From the results it showed the charge density and Model potential with the Percus Yevick structure factor can be used effectively to predict theoretically experimental values of electrical resistivity of any liquid metal.*

**Keywords:** *Electrical resistivity, Model potential, Structure factor, Form factor, Percus Yevick, Liquid metals.*

### 1.0 INTRODUCTION

Resistivity is the material property that pertains to how difficult it is for electrical current to flow through said materials. Materials with high resistivity are known as insulators while materials with low resistivity are known as conductors. Resistivity possesses the largest range of values for any physical property and it is essential in many materials applications including resistors in electrical circuits, dielectrics, resistive heating and superconducting.

The Ziman's nearly free electron (NFE) theory has been fairly successful in describing the quantitative

behavior of the electrical resistivity in simple liquid metals. This is because in these metals the mean free path is about one hundred times the interatomic distance and the weak scattering picture should be valid. Even for the heavy polyvalent metals (e.g. mercury, thallium and lead) where the mean free path is only about two interatomic distances, the NFE model can yield results, which are in reasonable agreement with experiments. Calculations of electrical resistivity using structure factor from various experiments or different versions of bare ion potential and dielectric function, gives correct order of magnitude but differ among themselves [1].

## 2.0 LITERATURE REVIEW

Baria [1] calculated the electrical resistivity of liquid metals using the structure factor derived via the charge hard sphere model with the model potential; results obtained are in agreement with experimental values of electrical resistivity.

Amah *et al.*, [2] calculated the electrical resistivity of liquid metals using the structure factor that depends on the ionic positions with Augmented Plane Wave method (APW) results obtained are in agreement with experimental values of electrical resistivity.

Aditya [3] calculated the electrical resistivity of liquid metals using Ashcroft empty core (EMC)[4] model pseudopotential results obtained are in agreement with experimental values of electrical resistivity.

Thakore *et al.*[5] calculated the electrical resistivity of liquid Lanthanides using a Model potential constructed by them results obtained are in agreement with experimental values.

In this present paper, the electrical resistivity of liquid metals will be calculated differently using the concept of charge density. The charge density already incorporated in the Model potential proposed by Pandya *et al.*[6] with the Percus-Yevick (PY) [7] structure factor is used to calculate electrical resistivity of liquid metals.

## 3.0 AIM AND RESEARCH PROBLEMS

The aim of this work is to calculate the electrical resistivity of liquid metals using Model pseudopotential method.

Research problems: to calculate the electrical resistivity of liquid metals in a simplified way and reduce cost of computation; to use an approach that will improve on previously calculated values of electrical resistivity of liquid metals by other researchers.

## 4.0 MATERIALS AND METHOD

The material that was used in this work are CodeBlocks C++ program package and computer. The model pseudo potential [6] in this work is given as:

$$W_{ion}(r) = \sum_{n=1}^2 B_n e^{-(r/na)} \quad r < R_c \quad (1)$$

the Pseudopotential of the electron has this form when it is inside the core radius  $R_c$  and  $r$  is the electron distance from the nucleus.

$$W_{ion}(r) = -\frac{Z_s}{r} \quad r > R_c \quad (2)$$

the Pseudo potential of the electron has this form when it is outside the core radius  $R_c$  and  $r$  is the electron distance from the nucleus.

The Fourier transform of the model pseudo potential in q-space is given as

$$W_{ion}(q) = 4\pi a^3 \rho \left[ \frac{B_1 H_1}{(1+a^2 q^2)^2} + \frac{8B_2 H_2}{(1+4a^2 q^2)^2} \right] - \frac{4\pi Z_s \rho}{q^2} \cdot \cos qR_c \quad (3)$$

where  $Z_s, a, q, \rho$  are the effective number of valence electrons per atom, measure of the softness of the repulsive potential, wave vector and charge density.  $B_1, B_2$  are coefficients of the Dirichlet series and  $H_1, H_2$  are represented as the sum of the repulsive and the oscillatory contributions. The charge density  $\rho$  is obtained from squaring the radial wave function and the radial wave function is obtained from the solution of the Schrodinger equation in spherical polar coordinate.

$$\rho = |R_{nl}(r)|^2 \quad (4)$$

where  $R_{nl}(r) = - \left[ \left( \frac{2Z}{na_0} \right)^3 \frac{(n-l-1)!}{2n \{(n+l)!\}^3} \right]^{\frac{1}{2}} e^{-\frac{\rho}{2}} \rho^l L_{n+l}^{2l+1}(\rho)$

where  $L_{n+l}^{2l+1}(\rho), n, l, a_0, \rho$  are the associated Laguerre polynomial, principal quantum number, angular momentum quantum number, Bohr radius and unitless quantity.

$$a_0 = \frac{4\pi \epsilon_0 \hbar^2}{\mu e^2}, \rho = \frac{2Zr}{na_0}$$

where  $r, \epsilon_0, \hbar, e$  are the electron distance from the nucleus, permittivity of free space, reduced Planck's constant and electronic charge.

$$B_1 = \frac{Z_s}{R_c} \left[ 1 - \frac{2a}{R_c} \right] e^{\frac{R_c}{a}}, B_2 = \frac{2Z_s}{R_c} \left[ \frac{a}{R_c} - 1 \right] e^{\frac{R_c}{2a}}$$

$$H_1 = 2 - e^{(Y_1)[Y_1(1+X_1)-(1-X_1)]} \times \frac{\sin qR_c}{aq} + [2 + Y_1(1+X_1)] \times \cos qR_c$$

$$H_2 = 2 - e^{(Y_2)[Y_2(1+X_2)-(1-X_2)]} \times \frac{\sin qR_c}{2aq} + [2 + Y_2(1+X_2)] \times \cos qR_c$$

where  $X_1 = a^2 q^2, X_2 = 2^2 a^2 q^2, Y_1 = \frac{R_c}{a}, Y_2 = \frac{R_c}{2a}$

The Thomas Fermi dielectric constant [8] is used to screen the form factor in this work and given as:

$$\epsilon(q) = 1 + \frac{k_o^2}{q^2}, k_o = 0.815k_f \left( \frac{r_s}{a_o} \right)^{\frac{1}{2}}$$

where  $k_f$  is Thomas-Fermi wave vector

The structure derived via the Percus Yevick Hard sphere (PHYS) model is one of the most straightforward and extensively used model and is obtained from the exact solution of the Percus-Yevick equation for Hard-Sphere (PY) diameter ( $\sigma$ ), which is obtained, to get best fit of  $S(q)$  with the experimental data. The PY integral equation to yield the structure factor using the expression as:

$$S(q\sigma) = \frac{1}{(1-nc(q\sigma))} \quad (5)$$

where  $\sigma$  is Hard-sphere diameter and the direct-correlation function  $c(q\sigma)$  in momentum space is yield as follows:

$$c(q\sigma) = -4\pi\sigma^3 \int_0^1 ds s^2 \frac{\sin(sq\sigma)}{sq} (\alpha + \beta s + \gamma s^2) \quad (6)$$

The parameter  $\alpha$ ,  $\beta$  and  $\gamma$  are the functions of packing density parameter  $\eta$ , the function of total fluid volume occupied by the sphere defined as:

$$\eta = \left(\frac{\pi}{6}\right) n\sigma^3, \quad \alpha = \left(\frac{(1+2\eta)^2}{(1-\eta)^4}\right), \quad \beta = \left(\frac{-6\eta\left(1+\frac{\eta}{2}\right)}{(1-\eta)^4}\right)$$

and 
$$\gamma = \left(\frac{\frac{1}{2}\eta(1+2\eta)^2}{(1-\eta)^4}\right)$$

On integrating the equation (6) over the volume of the sphere, the expression for the structure factor turns out to be:

$$S(q) = \left[ 1 + \left\{ \frac{24\eta}{(1-\eta)^4 y^6} \right\} \left\{ \begin{aligned} &(1+2\eta)^2 y^3 (\sin y - y \cos y) \\ &-6\eta \left(1 + \frac{\eta}{2}\right)^2 y^2 \{2y \sin y - (y^2 - 2) \cos y - 2\} \\ &+ \frac{\eta}{2} (1+2\eta)^2 \{ (4y^3 - 24y) \sin y - (y^2 - 12y^2 + 24) \cos y + 24 \} \end{aligned} \right\} \right]$$

In which  $y = q\sigma$

The liquid metal resistivity ( $\rho$ ) [9] is given as:

$$\rho = \left(\frac{4\pi^3 \hbar}{e^2}\right) \left(\frac{Z}{k_F}\right) \int_0^1 S(q) |W(q)|^2 q^3 dq \quad (7)$$

where  $S(q)$ ,  $\hbar$ ,  $e$ ,  $W(q)$  are the structure factor, reduced Planck's constant, electronic charge and screened form factor.

### 5.0 RESULT AND DISCUSSION

The input parameters used in the calculation are given in Table 1 and the comparison between the calculated values of electrical resistivity of liquid metals in this work and others previously calculated with the experimental values of electrical resistivity of liquid metals are given in Table 2. The experimental values of electrical resistivity gotten from Faber [10].

From table 2 calculated values of electrical resistivity are in perfect agreement with experimental values and have a higher accuracy when compared with Bari and Aditya values of electrical resistivity. Sodium (Na) and potassium (K) have the lowest resistivity values; meaning that they are very good conductors of heat and electric current while lead (Pb) and caesium (Cs) have the highest resistivity values indicating that they are very good insulators; heat and electric current do not pass through them easily.

**Table 1. Input Parameters Used in the Calculation of Electrical Resistivity**

Metals	Z	S(q)(a.u)	W(q)(a.u)	$K_f(a.u^{-1})$
Li	1	0.0509	-4.26	0.58
Na	1	0.0513	-2.40	0.48
K	1	0.0528	-2.49	0.39
Rb	1	0.8180	-2.55	0.36
Cs	1	0.0795	-3.16	0.33
Ag	1.5	0.0159	-5.55	0.66
Cd	2	0.2212	-1.91	0.74
Au	2	0.0210	-5.48	0.64
Mg	2	0.1959	-1.79	0.72
Al	3	0.4026	-1.10	0.91
Pb	4	0.4321	-1.71	0.83

**Table 2: Comparing calculated values with Bari, Aditya and experimental values of Electrical Resistivity (in  $\mu\Omega\text{cm}$ )**

Metals	Present work	Experimental value	Bari	Aditya
Li	24.61	24.70	21.15	38.76
Na	9.59	9.60	8.44	15.51
K	13.06	13.00	11.48	20.58
Rb	22.95	22.50	23.62	50.05
Cs	36.79	36.00	31.29	89.23
Ag	17.15	17.00	---	108.15
Cd	33.77	34.00	34.65	49.06
Au	30.66	31.00	---	439.93
Mg	26.97	26.00	29.13	37.93
Al	25.02	24.00	25.58	30.23
Pb	94.97	95.00	94.57	129.16

## 6.0. CONCLUSION

From the results obtained it showed that the charge density and Model potential with the Percus Yervick structure factor can be used effectively to predict theoretically experimental values of electrical resistivity of any liquid metal since it gives values which are in perfect agreement with experimental

values of electrical resistivity and has improved previous values calculated by other researchers

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