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# SECOND QUANTIZATION OF ELECTROMAGNETIC FIELD IN TERMS OF STRENGTHS $\vec{E}$ AND $\vec{H}$

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#### **ABSTRACT**

In the previous work, the Lagrange formalism for electromagnetic field in terms of strengths  $\vec{E}$  and  $\vec{H}$  has been elaborated. First, invariant Lagrange function for electromagnetic field has been written through electric and magnetic vectors  $\vec{E}$  and  $\vec{H}$ , and then, using Noether's theorem, expressions for dynamical variables (energy ,momentum, charge and spin)conserved in time have been written through vectors  $\vec{E}$  and  $\vec{H}$ . In this work, in development of the above ideas, we elaborated the second quantization of electromagnetic

field in terms of vectors  $\vec{E}$  and  $\vec{H}$ .

Keywords: Second quantization, electromagnetic field, strengths.

#### **INTRODUCTION**

In previous works, Maxwell's equations for electromagnetic field have been written in different forms. In particularly, they have been written through complex vector  $\vec{F} = \vec{E} + i\vec{H}$ . Furthermore, the Lagrange formalism for electromagnetic field in terms of complex isotropic vector  $\vec{F} = \vec{E} + i\vec{H}$  has been elaborated.

In this work, in development of the above ideas, we shall elaborate the second quantization of electromagnetic field in terms of strengths  $\vec{E}$  and  $\vec{H}$ .

#### **RESEARCH METHOD**

In the previous work, the Lagrange formalism for electromagnetic field in terms of strengths  $\vec{E}$  and  $\vec{H}$  has been elaborated. Invariant Lagrange function for electromagnetic field in terms of strengths  $\vec{E}$  and  $\vec{H}$  has been written.

Furthermore, using Noether's theorem, expressions for dynamical variables (energy, momentum, charge and spin) conserved in time have been obtained in terms of strengths  $\vec{E}$  and  $\vec{H}$ .

In this work, we shall develop the second quantization of electromagnetic field in terms of strengths  $\vec{E}$  and  $\vec{H}$ . Here, we shall find expressions for dynamical variables (energy, momentum, charge and spin) conserved in time in terms of the number of particles.

#### Second Quantization of Electromagnetic Field in Terms of Strengths $\vec{E}$ and $\vec{H}$

Let us introduce complex vector

$$\vec{F} = \vec{E} + i\vec{H},\tag{1}$$

where  $\vec{E},\vec{H}$  are electric and magnetic field strengths . Then, Maxwell's equations for vacuum

$$\begin{cases} \vec{\nabla} \times \vec{E} + \frac{\partial \vec{H}}{\partial t} = 0\\ \vec{\nabla} \times \vec{H} - \frac{\partial \vec{E}}{\partial t} = 0\\ \vec{\nabla} . \vec{E} = 0\\ \vec{\nabla} . \vec{H} = 0 \end{cases}$$
(2)

can be written in the form

$$\begin{cases} D^{0}\vec{F} = i\vec{D}\times\vec{F} \\ \vec{D}\vec{F} = 0 \end{cases}$$
(3)

Here

$$\mathbf{D}^0 = \frac{\mathbf{i}}{2} \frac{\partial}{\partial \mathbf{t}} \tag{4}$$

$$\vec{\mathbf{D}} = -\frac{\mathbf{i}}{2}\vec{\nabla} \tag{5}$$

The solution of equations (3) can be written in the form

$$\vec{F} = \vec{F}^0 e^{-2ikt + 2i\vec{k}\vec{r}},\tag{6}$$

Where

$$\vec{F}^{0} = \frac{i}{2} \begin{bmatrix} \sin \phi + i \operatorname{scos} \vartheta \cos \phi \\ - \cos \phi + i \operatorname{scos} \vartheta \sin \phi \\ - i \operatorname{ssin} \vartheta \end{bmatrix},$$
(7)

 $k = |\vec{k}|$  is the energy,  $s = \pm 1$  is the polarization and  $\varphi, \vartheta$  are the angles of orientation of the wave vector  $\vec{k}$ , chosen so that  $k_1 + ik_2 = k \sin \vartheta e^{i\varphi}$ ,  $k_3 = k \cos \vartheta$ .

Separating real and imaginary parts in formula (7), we obtain the solution for field strengths

$$\vec{E} = \frac{1}{2} \begin{bmatrix} \sin \varphi \\ -\cos \varphi \\ 0 \end{bmatrix} e^{-2ikt + 2i\vec{k}\vec{r}}$$
(8)  
$$\vec{H} = \frac{1}{2} \begin{bmatrix} \sin \varphi \\ \cos \varphi \\ \sin \varphi \\ -\sin \varphi \end{bmatrix} e^{-2ikt + 2i\vec{k}\vec{r}} .$$
(9)

In previous work, it has been proved that Maxwell's equations (3) can be obtained from the Lagrange function

$$\mathbf{L} = \frac{\mathbf{i}}{2} \left\{ \left[ \mathbf{D}^0 \vec{\mathbf{F}} - \mathbf{i} \vec{\mathbf{D}} \times \vec{\mathbf{F}} \right] \vec{\mathbf{F}}^* - \left[ \mathbf{D}^0 \vec{\mathbf{F}}^* + \mathbf{i} \vec{\mathbf{D}} \times \vec{\mathbf{F}}^* \right] \vec{\mathbf{F}} \right\} / \left( \frac{\vec{\mathbf{F}} \vec{\mathbf{F}}^*}{2} \right)^{1/2}.$$
(10)

In terms of strengths  $\vec{E}$  and  $\vec{H}$ , formula (10) can be written in the form

$$L = \frac{i}{4|\vec{E}|} \left\{ \left[ \vec{E} \frac{\partial \vec{H}}{\partial t} - \vec{H} \frac{\partial \vec{E}}{\partial t} + \vec{E} \cdot \vec{\nabla} \times \vec{E} + \vec{H} \cdot \vec{\nabla} \times \vec{H} \right] + i \left[ \vec{E} \frac{\partial \vec{E}}{\partial t} + \vec{H} \frac{\partial \vec{H}}{\partial t} - \vec{E} \cdot \vec{\nabla} \times \vec{H} + \vec{H} \cdot \vec{\nabla} \times \vec{E} \right] \right\}. (11)$$

Using Noether's theorem, from the Lagrange function (11), we find the following expressions for dynamical variables (energy, momentum, charge and spin) conserved in time.

For energy we find

$$\mathbf{E} = \int \mathbf{T}^{00} \mathbf{d}^3 \mathbf{x},\tag{12}$$

Where

$$T^{00} = \frac{1}{4|\vec{E}|} \left[ -\vec{H} \frac{\partial \vec{E}}{\partial t} + i\vec{E} \frac{\partial \vec{E}}{\partial t} + \vec{E} \frac{\partial \vec{H}}{\partial t} + i\vec{H} \frac{\partial \vec{H}}{\partial t} \right].$$
(13)

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Considering the solution in the form of plane waves

$$\vec{E} = \vec{E}^0 e^{-2ikt+2i\vec{k}\vec{r}}.$$
(14)

$$\vec{H} = \vec{H}^0 e^{-2ikt+2i\vec{k}\vec{r}},$$
(15)

we obtain

$$\mathbf{T}^{00} = \mathbf{k} \big| \vec{\mathbf{E}} \big|. \tag{16}$$

Similarly for momentum, we have

$$\mathbf{P}^{\mathbf{j}} = \int \mathbf{T}^{0\mathbf{j}} \mathbf{d}^3 \mathbf{x},\tag{17}$$

Where

$$T^{0j} = \frac{1}{4|\vec{E}|} \left[ -\vec{H} \frac{\partial \vec{E}}{\partial x_j} + i\vec{E} \frac{\partial \vec{E}}{\partial x_j} + \vec{E} \frac{\partial \vec{H}}{\partial x_j} + i\vec{H} \frac{\partial \vec{H}}{\partial x_j} \right].$$
(18)

With the relations (14)-(15), we obtain

$$\mathbf{T}^{0j} = \mathbf{k}_j \big| \vec{\mathbf{E}} \big|. \tag{19}$$

For charge, we find

$$Q = \int j^0 d^3 x, \qquad (20)$$

Where

$$j^{0} = -\frac{i}{|\vec{E}|} \left( i\vec{E}\vec{E} + i\vec{H}\vec{H} \right), \tag{21}$$

$$\mathbf{j}^0 = |\vec{\mathbf{E}}|. \tag{22}$$

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In the same way, for the spin pseudo vector we find

$$\vec{s} = \frac{\vec{E} \times \vec{H}}{|\vec{E}|}.$$
(23)

Let us expand the wave function  $\vec{E}(x)$  and  $\vec{H}(x)$  in Fourier series

$$\vec{E}(x) = \sum_{k,s} a_s(\vec{k}) \vec{E}_s(\vec{k}) e^{-2ikt + 2i\vec{k}\vec{r}}, \qquad (24)$$

$$\vec{E}^{*}(x) = \sum_{k,s} a_{s}^{*}(\vec{k}) \vec{E}_{s}^{*}(\vec{k}) e^{2ikt - 2i\vec{k}\vec{r}}.$$
(25)

$$\vec{H}(x) = \sum_{k,s} a_s(\vec{k}) \vec{H}_s(\vec{k}) e^{-2ikt + 2i\vec{k}\vec{r}}, \qquad (26)$$

$$\vec{\mathrm{H}}^{*}(\mathrm{x}) = \sum_{\mathrm{k},\mathrm{s}} \mathrm{a}_{\mathrm{s}}^{*}(\vec{\mathrm{k}}) \vec{\mathrm{H}}_{\mathrm{s}}^{*}(\vec{\mathrm{k}}) \mathrm{e}^{2\mathrm{i}\mathrm{k}\mathrm{t}-2\mathrm{i}\vec{\mathrm{k}}\vec{\mathrm{r}}}.$$
(27)

Replacing formulas (24)- (19) in formulas (12), (17), (20) and (23) and considering the normalization condition

$$\int \left[ \frac{\vec{E}_{ks}^* \vec{E}_{k's'}}{2|\vec{E}|} + \frac{\vec{H}_{ks}^* \vec{H}_{k's'}}{2|\vec{E}|} \right] d^3 x = \delta_{kk'} \delta_{ss'}, \tag{28}$$

we obtain

$$\mathbf{E} = \sum_{\mathbf{k},s} \mathbf{k} \left[ \mathbf{a}_{s}^{*}(\vec{\mathbf{k}}) \mathbf{a}_{s}(\vec{\mathbf{k}}) \right], \tag{29}$$

$$P_{j} = \sum_{k,s} k_{j} \left[ a_{s}^{*}(\vec{k}) a_{s}(\vec{k}) \right], \qquad (30)$$

$$Q = \sum_{k,s} \left[ a_s^*(\vec{k}) a_s(\vec{k}) \right], \tag{31}$$

$$S_{j} = \sum_{k,s} \mathfrak{w}_{j} \left[ a_{s}^{*}(\vec{k}) a_{s}(\vec{k}) \right].$$
(32)

Here  $\vec{a}$  is a unit vector in the direction of the polarization vector  $\vec{S}$ .

Changing  $a_s(\vec{k})$  into operator  $\hat{a}_s(\vec{k})$ , we find the following operators for physical quantities

$$\widehat{\mathbf{E}} = \sum_{\mathbf{k},s} \mathbf{k} [\widehat{\mathbf{a}}_{s}^{+} (\vec{\mathbf{k}}) \widehat{\mathbf{a}}_{s} (\vec{\mathbf{k}})], \qquad (33)$$

$$\widehat{\mathbf{P}}_{j} = \sum_{\mathbf{k},s} \mathbf{k}_{j} [\widehat{\mathbf{a}}_{s}^{+}(\vec{\mathbf{k}}) \widehat{\mathbf{a}}_{s}(\vec{\mathbf{k}})], \qquad (34)$$

$$\widehat{\mathbf{Q}} = \sum_{\mathbf{k},s} [\widehat{\mathbf{a}}_{s}^{+}(\vec{\mathbf{k}})\widehat{\mathbf{a}}_{s}(\vec{\mathbf{k}})], \qquad (35)$$

$$\hat{\mathbf{S}}_{j} = \sum_{\mathbf{k},s} \mathbf{x}_{j} [\hat{\mathbf{a}}_{s}^{+}(\vec{\mathbf{k}}) \hat{\mathbf{a}}_{s}(\vec{\mathbf{k}})].$$
(36)

To ensure the positive determination of energy, we must require the following commutation relations

$$\left[\hat{a}_{s}^{+}(\vec{k}), \hat{a}_{s'}(\vec{k}')\right]_{-} = \delta_{kk'}\delta_{ss'}.$$
(37)

Using formulas (33)-(36), we find expressions for eigenvalues of the above operators

$$\mathbf{E} = \sum_{\mathbf{k},\mathbf{s}} \mathbf{k}[\mathbf{N}_{\mathbf{k}\mathbf{s}}],\tag{38}$$

$$P_{j} = \sum_{k,s} k_{j} [N_{ks}], \qquad (39)$$

$$\mathbf{D} = \sum_{\mathbf{k},\mathbf{s}} [\mathbf{N}_{\mathbf{k}\mathbf{s}}],\tag{40}$$

$$S_{i} = \sum_{k,s} \alpha_{i} [N_{ks}]. \tag{41}$$

Here  $N_{ks}$  is the number of particles.

#### **DISCUSSION AND CONCLUSION**

In previous works, Maxwell's equations for electromagnetic field have been written through complex isotropic vectors  $\vec{F} = \vec{E} + i\vec{H}$  and the Lagrange formalism for electromagnetic field in terms of complex isotropic vectors  $\vec{F} = \vec{E} + i\vec{H}$  has been elaborated. Furthermore, the second quantization of electromagnetic field in terms of complex isotropic vectors  $\vec{F} = \vec{E} + i\vec{H}$  has been developed. In this work, we went further and we elaborated the second quantization of electromagnetic field in terms of strengths  $\vec{E}$  and  $\vec{H}$ . Here, dynamical variables (energy, momentum, charge and spin) conserved in time have been expressed through the number of particles.

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