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SECOND QUANTIZATION OF ELECTROMAGNETIC FIELD IN TERMS OF STRENGTHS $\vec{E}$ AND $\vec{H}$

DR. BULIKUNZIRA SYLVESTRE*

*University of Rwanda, University Avenue, B.P117, Butare, Rwanda

ABSTRACT

In the previous work, the Lagrange formalism for electromagnetic field in terms of strengths $\vec{E}$ and $\vec{H}$ has been elaborated. First, invariant Lagrange function for electromagnetic field has been written through electric and magnetic vectors $\vec{E}$ and $\vec{H}$, and then, using Noether's theorem, expressions for dynamical variables (energy ,momentum, charge and spin)conserved in time have been written through vectors $\vec{E}$ and $\vec{H}$.

In this work, in development of the above ideas, we elaborated the second quantization of electromagnetic field in terms of vectors $\vec{E}$ and $\vec{H}$.

***Keywords:* Second quantization, electromagnetic field, strengths.**

INTRODUCTION

In previous works, Maxwell's equations for electromagnetic field have been written in different forms. In particularly, they have been written through complex vector $\vec{F} = \vec{E} + i\vec{H}$. Furthermore, the Lagrange formalism for electromagnetic field in terms of complex isotropic vector $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated.

In this work, in development of the above ideas, we shall elaborate the second quantization of electromagnetic field in terms of strengths $\vec{E}$ and $\vec{H}$.

RESEARCH METHOD

In the previous work, the Lagrange formalism for electromagnetic field in terms of strengths $\vec{E}$ and $\vec{H}$ has been elaborated. Invariant Lagrange function for electromagnetic field in terms of strengths $\vec{E}$ and $\vec{H}$ has been written.

Furthermore, using Noether's theorem, expressions for dynamical variables (energy, momentum, charge and spin) conserved in time have been obtained in terms of strengths $\vec{E}$ and $\vec{H}$.

In this work, we shall develop the second quantization of electromagnetic field in terms of strengths $\vec{E}$ and $\vec{H}$. Here, we shall find expressions for dynamical variables (energy, momentum, charge and spin) conserved in time in terms of the number of particles.

Second Quantization of Electromagnetic Field in Terms of Strengths $\vec{E}$ and $\vec{H}$

Let us introduce complex vector

$$\vec{F} = \vec{E} + i\vec{H}, \qquad (1)$$

where $\vec{E}, \vec{H}$ are electric and magnetic field strengths . Then, Maxwell's equations for vacuum

$$\begin{cases} \vec{\nabla} \times \vec{E} + \frac{\partial \vec{H}}{\partial t} = 0 \\ \vec{\nabla} \times \vec{H} - \frac{\partial \vec{E}}{\partial t} = 0 \\ \vec{\nabla} . \vec{E} = 0 \\ \vec{\nabla} . \vec{H} = 0 \end{cases} \qquad (2)$$

can be written in the form

$$\begin{cases} D^0 \vec{F} = i\vec{D} \times \vec{F} \\ \vec{D}\vec{F} = 0 \end{cases} \qquad (3)$$

Here

$$D^0 = \frac{i}{2}\frac{\partial}{\partial t} \qquad (4)$$

$$\vec{D} = -\frac{i}{2}\vec{\nabla} \qquad (5)$$

The solution of equations (3) can be written in the form

$$\vec{F} = \vec{F}^0 e^{-2ikt + 2i\vec{k}\vec{r}}, \qquad (6)$$

Where

$$\vec{F}^0 = \frac{i}{2}\begin{bmatrix} \sin\varphi + is\cos\vartheta\cos\varphi \\ -\cos\varphi + is\cos\vartheta\sin\varphi \\ -is\sin\vartheta \end{bmatrix}, \qquad (7)$$

$k = |\vec{k}|$ is the energy, $s = \pm 1$ is the polarization and φ, ϑ are the angles of orientation of the wave vector $\vec{k}$, chosen so that $k_1 + ik_2 = k\sin\vartheta\, e^{i\varphi}$, $k_3 = k\cos\vartheta$.

Separating real and imaginary parts in formula (7), we obtain the solution for field strengths

$$\vec{E} = \frac{1}{2}\begin{bmatrix} \sin\varphi \\ -\cos\varphi \\ 0 \end{bmatrix} e^{-2ikt+2i\vec{k}\vec{r}} \qquad (8)$$

$$\vec{H} = \frac{1}{2}\begin{bmatrix} s\cos\vartheta\cos\varphi \\ s\cos\vartheta\sin\varphi \\ -s\sin\vartheta \end{bmatrix} e^{-2ikt+2i\vec{k}\vec{r}} \quad . \qquad (9)$$

In previous work, it has been proved that Maxwell's equations (3) can be obtained from the Lagrange function

$$L = \frac{i}{2}\{[D^0\vec{F} - i\vec{D}\times\vec{F}]\vec{F}^* - [D^0\vec{F}^* + i\vec{D}\times\vec{F}^*]\vec{F}\}/\left(\frac{\vec{F}\vec{F}^*}{2}\right)^{1/2}. \quad (10)$$

In terms of strengths $\vec{E}$ and $\vec{H}$, formula (10) can be written in the form

$$L = \frac{i}{4|\vec{E}|}\left\{\left[\vec{E}\frac{\partial\vec{H}}{\partial t} - \vec{H}\frac{\partial\vec{E}}{\partial t} + \vec{E}.\vec{\nabla}\times\vec{E} + \vec{H}.\vec{\nabla}\times\vec{H}\right] + i\left[\vec{E}\frac{\partial\vec{E}}{\partial t} + \vec{H}\frac{\partial\vec{H}}{\partial t} - \vec{E}.\vec{\nabla}\times\vec{H} + \vec{H}.\vec{\nabla}\times\vec{E}\right]\right\}. \quad (11)$$

Using Noether's theorem, from the Lagrange function (11), we find the following expressions for dynamical variables (energy, momentum, charge and spin) conserved in time.

For energy we find

$$E = \int T^{00} d^3x, \qquad (12)$$

Where

$$T^{00} = \frac{1}{4|\vec{E}|}\left[-\vec{H}\frac{\partial\vec{E}}{\partial t} + i\vec{E}\frac{\partial\vec{E}}{\partial t} + \vec{E}\frac{\partial\vec{H}}{\partial t} + i\vec{H}\frac{\partial\vec{H}}{\partial t}\right]. \qquad (13)$$

Considering the solution in the form of plane waves

$$\vec{E} = \vec{E}^0 e^{-2ikt+2i\vec{k}\vec{r}}, \qquad (14)$$

$$\vec{H} = \vec{H}^0 e^{-2ikt+2i\vec{k}\vec{r}}, \qquad (15)$$

we obtain

$$T^{00} = k|\vec{E}|. \qquad (16)$$

Similarly for momentum, we have

$$P^j = \int T^{0j} d^3x, \qquad (17)$$

Where

$$T^{0j} = \frac{1}{4|\vec{E}|}\left[-\vec{H}\frac{\partial\vec{E}}{\partial x_j} + i\vec{E}\frac{\partial\vec{E}}{\partial x_j} + \vec{E}\frac{\partial\vec{H}}{\partial x_j} + i\vec{H}\frac{\partial\vec{H}}{\partial x_j}\right]. \qquad (18)$$

With the relations (14)-(15), we obtain

$$T^{0j} = k_j|\vec{E}|. \qquad (19)$$

For charge, we find

$$Q=\int j^0 d^3x, \qquad (20)$$

Where

$$j^0 = -\frac{i}{|\vec{E}|}\left(i\vec{E}\vec{E} + i\vec{H}\vec{H}\right), \qquad (21)$$

Or

$$j^0 = |\vec{E}|. \qquad (22)$$

In the same way, for the spin pseudo vector we find

$$\vec{s} = \frac{\vec{E}\times\vec{H}}{|\vec{E}|}. \qquad (23)$$

Let us expand the wave function $\vec{E}(x)$ and $\vec{H}(x)$ in Fourier series

$$\vec{E}(x) = \sum_{k,s} a_s(\vec{k})\vec{E}_s(\vec{k})e^{-2ikt+2i\vec{k}\vec{r}}, \qquad (24)$$

$$\vec{E}^*(x) = \sum_{k,s} a_s^*(\vec{k})\vec{E}_s^*(\vec{k})e^{2ikt-2i\vec{k}\vec{r}}. \qquad (25)$$

$$\vec{H}(x) = \sum_{k,s} a_s(\vec{k})\vec{H}_s(\vec{k})e^{-2ikt+2i\vec{k}\vec{r}}, \qquad (26)$$

$$\vec{H}^*(x) = \sum_{k,s} a_s^*(\vec{k})\vec{H}_s^*(\vec{k})e^{2ikt-2i\vec{k}\vec{r}}. \qquad (27)$$

Replacing formulas (24)- (19) in formulas (12), (17), (20) and (23) and considering the normalization condition

$$\int\left[\frac{\vec{E}_{ks}^*\vec{E}_{k's'}}{2|\vec{E}|} + \frac{\vec{H}_{ks}^*\vec{H}_{k's'}}{2|\vec{E}|}\right]d^3x = \delta_{kk'}\delta_{ss'}, \qquad (28)$$

we obtain

$$E = \sum_{k,s} k[a_s^*(\vec{k})a_s(\vec{k})], \qquad (29)$$

$$P_j = \sum_{k,s} k_j[a_s^*(\vec{k})a_s(\vec{k})], \qquad (30)$$

$$Q = \sum_{k,s}[a_s^*(\vec{k})a_s(\vec{k})], \qquad (31)$$

$$S_j = \sum_{k,s} æ_j[a_s^*(\vec{k})a_s(\vec{k})]. \qquad (32)$$

Here $\vec{æ}$ is a unit vector in the direction of the polarization vector $\vec{S}$.

Changing $a_s(\vec{k})$ into operator $\hat{a}_s(\vec{k})$, we find the following operators for physical quantities

$$\hat{E} = \sum_{k,s} k[\hat{a}_s^+(\vec{k})\hat{a}_s(\vec{k})], \qquad (33)$$

$$\hat{P}_j = \sum_{k,s} k_j[\hat{a}_s^+(\vec{k})\hat{a}_s(\vec{k})], \qquad (34)$$

$$\hat{Q} = \sum_{k,s}[\hat{a}_s^+(\vec{k})\hat{a}_s(\vec{k})], \qquad (35)$$

$$\hat{S}_j = \sum_{k,s} æ_j[\hat{a}_s^+(\vec{k})\hat{a}_s(\vec{k})]. \qquad (36)$$

To ensure the positive determination of energy, we must require the following commutation relations

$$[\hat{a}_s^+(\vec{k}), \hat{a}_{s'}(\vec{k}')]_- = \delta_{kk'}\delta_{ss'}. \qquad (37)$$

Using formulas (33)-(36), we find expressions for eigenvalues of the above operators

$$E = \sum_{k,s} k[N_{ks}], \qquad (38)$$

$$P_j = \sum_{k,s} k_j [N_{ks}], \qquad (39)$$

$$Q = \sum_{k,s} [N_{ks}], \qquad (40)$$

$$S_j = \sum_{k,s} æ_j [N_{ks}]. \qquad (41)$$

Here N_{ks} is the number of particles.

DISCUSSION AND CONCLUSION

In previous works, Maxwell's equations for electromagnetic field have been written through complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$ and the Lagrange formalism for electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$ has been elaborated. Furthermore, the second quantization of electromagnetic field in terms of complex isotropic vectors $\vec{F} = \vec{E} + i\vec{H}$ has been developed. In this work, we went further and we elaborated the second quantization of electromagnetic field in terms of strengths $\vec{E}$ and $\vec{H}$. Here, dynamical variables (energy, momentum, charge and spin) conserved in time have been expressed through the number of particles.

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