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TO FIND THE COMPARISON OF DIAMETER AND DENSITY OF AN INTERVAL GRAPH TOWARDS DOMINATING SETS

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ABSTRACT:

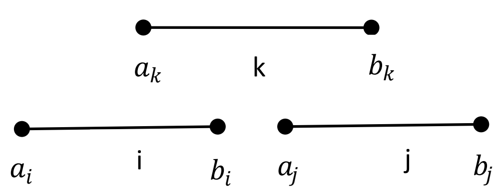
In this paper we investigate the problem of "the comparison of diameter and density of an interval graph towards dominating sets". In interval graphs have wide range of applications in the field of scheduling, computer networking, genetics etc. The diameter and density of a graph are fundamental topological parameters that have many practical applications in real world networks. For this purpose, certain classes of interval graphs are taken into consideration and their diameter, density towards dominating sets are obtained.

***Key words:* Interval family, Interval graph, Dominating set, Distance, Diameter and Density.**

INTRODUCTION

A finite number of intervals on a straight line, a graph associated with this set of intervals can be constructed in the following manner that is- each interval corresponds to a vertex of the graph and two vertices are connected by an edge if and only if the corresponding intervals overlap atleast partially.

Let $I = \{I_{1,} I_{2,} I_{3,}.\dots..I_n\}$ be an interval family. Each interval $I_{i,}$ in I is represented by $[a_i\ ,b_i]$ for i = 1,2,3,….,n. Here a_i is called left end and b_i is called right end of the interval i . Without loss of generality, we may assume that there are 2n end points which are distinct. The intervals are labeled in the increasing of their right end points.



The above interval family is labeled as I = $[a_i\ ,b_i]$, j= $[a_j\ ,b_j]$ and k = $[a_k\ ,b_k]$ whre I < j < k.

A graph G = (V, E) is an interval graph, if the vertex set V can be put into one to one correspondence with a set of intervals *I* on the real line R such that two vertices are adjacent in G if and only if their corresponding intervals have non-empty intersection. The set *I* is called an interval representation of G and G is referred to as the intersection graph *I*.

The research of the domination in graphs has been ever green of the graph theory. Its basic concepts are the dominating set and the domination number. The theory of domination in graphs was introduced by Ore [1] and Berge[2]. A survey on result and application of dominating set was presented by E.J.Cockayane and S.T.Hedetniemi [3]. In 1997 V.R.Kulliet. all introduced the concept non-split domination [4]. The diameter and radius are two of the most basic graph parameters. The diameter of a graph is the largest distance between its vertices. The distance orientation of graphs by V. Chvatal [5], the diameter of a graph G is the maximum eccentricity of all its vertices and is denoted by diam(G)- that is Diam(G) = Max$\{e(u): u \subset V(G)\}$, where the maximum distance from a vertex u to any vertex v of G is called eccentricity of the vertex u is denoted by e(u)- that is $e(u) =$ = Max$\{d(u, v): v \in V(G)\}$, where as the distance between two vertices u *and* v of a graph is the length of the shortest path between them and is denoted by $d_G(u, v)$ or $d(u, v)$.

The density of G is the ratio of edges in G to the maximum possible number of edges. Let G be a graph. A subset D of V is called a dominating set if, every vertex in V-D is adjacent to some vertex in D. The distance $d(u, v)$ from a vertex u to a vertex v in a graph G is the length of a shortest $u - v$ walk if one exist. Otherwise $d(u, v) = \infty$.

DENSITY AND AVERAGE DEGREE

The density of a graph G = (V,E) measures how many edges are in the set E compared to the maximum possible number of edges between vertices in the set of V. Density of a graph is calculated as follows:

. As undirected graph has no loops and can have at most $\frac{|V|\,(|V-1|)}{2}$ edges. So the density of an undirected graph is $\frac{2\,|E|}{|V|(|V-1|)}$.

. A directed graph has no loops and can have at most $|V|(|V-1|)$ edges. So the density of a directed graph is $\frac{|E|}{|V|(|V-1|)}$.

The average degree of a graph G is another measure of how many edges are in the set E compared to the number of vertices in the set V because, each edge is incident to two vertices and counts in the degree of both vertices. The average degree of an undirected graph is $\frac{2\,|E|}{|V|}$.

Density $d = \frac{\bar{d}}{|V|}$, where $\bar{d}$ = Average degree.

Theorem1: Let G be a contained interval graph corresponding to an interval family *I*, i ∊ D, j ≠ 1, then $|NSDS| \geq dia(G) > \frac{\bar{d}}{n-1}$ (or) $dia(G) \geq |NSDS| > \frac{\bar{d}}{n-1}$.

Proof:- Let d be a dominating set of the given interval graph G corresponding to a contained interval family *I*. If i and j are any two intervals in *I* such that , i ∊ DS, , j ≠ 1, and j is contained in i and if there is atleast one interval to the left of j that intersects j and atleast one interval k ≠ i, to the right of j that intersects j. Then we have to show that the non-split domination occurs in G.

Suppose there is at least one interval k ≠ i to the right of j that intersects j. Then it is obvious that j is adjacent to k in $\langle V - D\rangle$, so that there will not be any disconnection $\langle V - D\rangle$. Since there is at least one interval to the left of j that intersects j, there will not be any disconnection in$\langle V - D\rangle$, to its left. Thus, we get non-spilt domination in G, it is denoted by NSDS.

Again we have to prove that the diameter of G corresponding to an interval family I = $\{1,2,3 \ldots\ldots i, j, \ldots n\}$.

First we will discuss the distances of G. For any two vertices i, j in an interval graph G corresponding to an internal family I, the distance from i *to* j is denoted by d(i,j) and defined as the length of a shortest i–j in an interval graph. The term 'distance' is just defined by satisfying all four of the following properties.

i) $d(i,j) \geq 0, \ \forall \ i,j \in V(G)$

ii) $d(i,j) = 0 \quad iff \ \ i = j$

iii) $d(i,j) = d(j,i)\ , \forall \ i,j \in V(G)$

iv) $d(i,k) \leq d(i,j) + d(j,k)\ , \forall \ i,j,k \in V(G)$

Suppose an interval graph V(G) is not a connected graph and G_1 and G_2 are two graphs of G such that $G = G_1 \cup G_2$ and $E(G_1) \cup E(G_2) = E(G)$ and $G_1 \cap G_2 = \emptyset$ that implies $V(G_1) \cap V(G_2) = \emptyset$ and $E(G_1) \cap E(G_2)$

= $\emptyset$. Then $d(i,j) = \infty$ for i∈ V(G_1) and j $\in V(G_2)$. This contradicts that i-j is shortest. Therefore G must be connected.

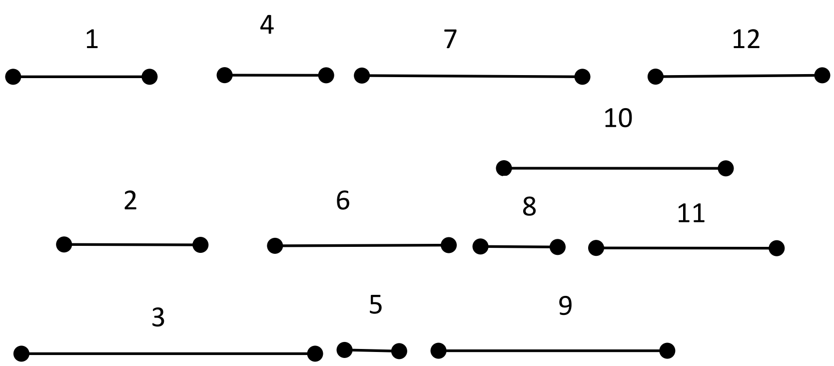
The diameter and radius are two of the most basic graph parameters. The diameter of a graph is the largest distance between the vertices. The distance orientations of graph by V.Chvatal[5], the diameter of a graph G is the maximum eccentricity of all its vertices and is denoted by dia(G), that is dia(G) = max$\{ecc(v) : v \in V(G)\}$, where the maximum distance from vertex v to any vertex of G is called 'eccentricity' of the vertex and is denoted by ecc(v), that is ecc(v) = max$\{d(v,u) : u \in V(G)\}$, where as the distance $d(v,u)$ between two vertices v *and* u of a graph is the length of the shortest path between them and is denoted by $d(v,u)$.

Finally we will find a density d of G corresponding to an interval family $I = \{I_{1,} I_{2,} I_{3,} \ldots\ldots I_n\}$. We find the degree of vertices $\deg(1), \deg(2) \ldots\ldots\ldots, \deg(n)$ in G corresponding to I. that is we discuss $\frac{2E}{n(n-1)}$, where n = number of vertices and E = number of edges.

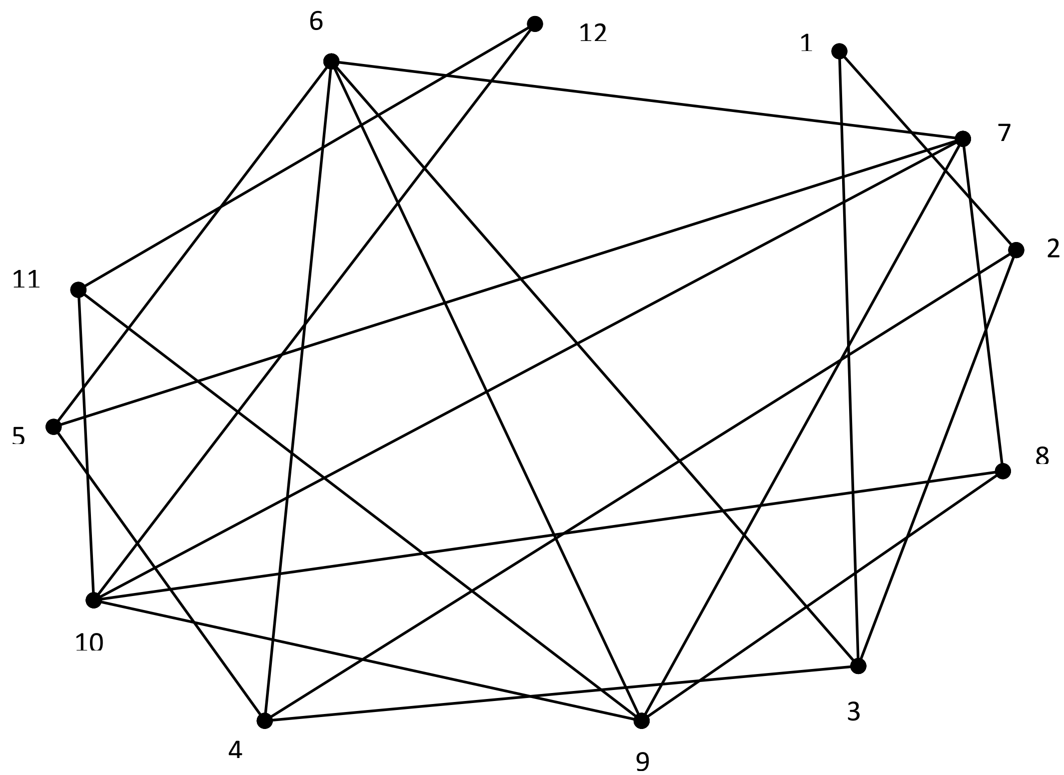
First we find the average degree of G. Suppose; an interval graph G has n vertices with degrees $\deg(1), \deg(2) \ldots\ldots\ldots \deg(n)$. Add together all vertices to get a new number D= 2E, where E= number of edges in G. In words, for any graph the sum of the degrees of the vertices equals to twice the number of edges. Therefore, we we find the average degree which can be denoted by $\bar{d} = \frac{\deg(1)+\deg(2)+\cdots..deg(n)}{n}$. Finally, we find the density of G corresponding to I. So that $d = \frac{\bar{d}}{n-1} = \frac{D}{n(n-1)} = \frac{2E}{n(n-1)}$.

From the above proof we can deduce the conditions which are $dia(G) \geq |NSDS| \geq \frac{\bar{d}}{n-1}$.
Hence the proof.

Illustration 1:



Interval Family (I.F.)



Interval Graph (I.G.)- *G*

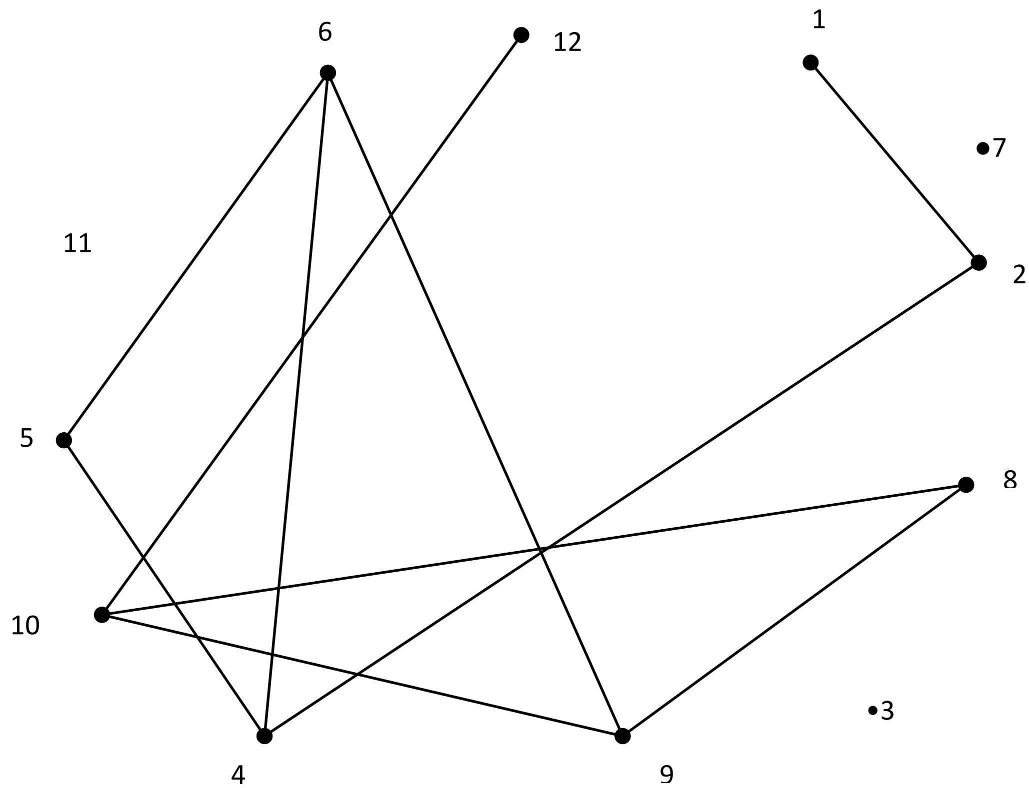
To find dominating set of G:

$I = \{1,2,3,4,5,6,7,8,9,10,11,12\}$,

Dominating set = {3,7,11}

$\therefore |DS| = 3$

$V - D = \{1,2,4,5,6,8,9,10,12\}$



Interval Graph (I.G.) − G'

From the picture above it is understood that the interval induced graph G' is connected. The nonsplit dominating set (NSDS) of G' is $\{1,4,10\}$. $\therefore$ $|NSDS| = 3$.

To find the average degree of G for density:

deg(1)=2, deg(2)=3, deg(3)=4, deg(4)=4, deg(5)=3, deg(6)=5, deg(7)=5, deg(8)=3, deg(9)=5, deg(10)=5, deg(11)=3, deg(12)=2.

$\therefore$ The sum of the degrees of vertices $D = \sum_{i=1}^{12} deg(i)$ = 44.

Average degree $\overline{D} = \frac{\sum_{i=1}^{12} \deg(i)}{12} = \frac{44}{12} = 3.7$

$\therefore$ Density $d = \frac{2E}{n(n-1)} = \frac{44}{12X11} = \frac{4}{12} = 0.33$

To find the diameter of $G = max\{e(i) : i \in V(G)\}$

The diameter of vertices $d(i,j)$ are as follows, whwre $i,j \in V(G)$

$d(1,1) = 0$	$d(2,1) = 1$	$d(3,1) = 1$	$d(4,1) = 2$
$d(1,2) = 1$	$d(2,2) = 0$	$d(3,2) = 1$	$d(4,2) = 1$
$d(1,3) = 1$	$d(2,3) = 1$	$d(3,3) = 0$	$d(4,3) = 1$
$d(1,4) = 2$	$d(2,4) = 1$	$d(3,4) = 1$	$d(4,4) = 0$
$d(1,5) = 3$	$d(2,5) = 2$	$d(3,5) = 2$	$d(4,5) = 1$
$d(1,6) = 2$	$d(2,6) = 2$	$d(3,6) = 2$	$d(4,6) = 1$
$d(1,7) = 3$	$d(2,7) = 3$	$d(3,7) = 2$	$d(4,7) = 2$
$d(1,8) = 4$	$d(2,8) = 4$	$d(3,8) = 3$	$d(4,8) = 3$
$d(1,9) = 3$	$d(2,9) = 4$	$d(3,9) = 2$	$d(4,9) = 2$
$d(1,10) = 4$	$d(2,10) = 4$	$d(3,10) = 3$	$d(4,10) = 3$
$d(1,11) = 4$	$d(2,11) = 4$	$d(3,11) = 3$	$d(4,11) = 3$
$d(1,12) = 5$	$d(2,12) = 5$	$d(3,12) = 4$	$d(4,12) = 3$

$d(5,1) = 3$	$d(6,1) = 2$	$d(7,1) = 3$	$d(8,1) = 4$
$d(5,2) = 2$	$d(6,2) = 2$	$d(7,2) = 3$	$d(8,2) = 4$
$d(5,3) = 2$	$d(6,3) = 3$	$d(7,3) = 2$	$d(8,3) = 3$
$d(5,4) = 1$	$d(6,4) = 1$	$d(7,4) = 2$	$d(8,4) = 3$
$d(5,5) = 0$	$d(6,5) = 2$	$d(7,5) = 1$	$d(8,5) = 2$
$d(5,6) = 1$	$d(6,6) = 0$	$d(7,6) = 1$	$d(8,6) = 2$
$d(5,7) = 1$	$d(6,7) = 1$	$d(7,7) = 0$	$d(8,7) = 1$
$d(5,8) = 2$	$d(6,8) = 2$	$d(7,8) = 1$	$d(8,8) = 0$
$d(5,9) = 2$	$d(6,9) = 2$	$d(7,9) = 1$	$d(8,9) = 1$
$d(5,10) = 2$	$d(6,10) = 2$	$d(7,10) = 1$	$d(8,10) = 1$
$d(5,11) = 3$	$d(6,11) = 2$	$d(7,11) = 2$	$d(8,11) = 2$
$d(5,12) = 3$	$d(6,12) = 3$	$d(7,12) = 2$	$d(8,12) = 2$

$d(9,1) = 3$	$d(10,1) = 4$	$d(11,1) = 4$	$d(12,1) = 5$
$d(9,2) = 4$	$d(10,2) = 4$	$d(11,2) = 4$	$d(12,2) = 5$
$d(9,3) = 2$	$d(10,3) = 3$	$d(11,3) = 3$	$d(12,3) = 4$
$d(9,4) = 2$	$d(10,4) = 3$	$d(11,4) = 3$	$d(12,4) = 3$
$d(9,5) = 2$	$d(10,5) = 2$	$d(11,5) = 3$	$d(12,5) = 3$
$d(9,6) = 2$	$d(10,6) = 2$	$d(11,6) = 2$	$d(12,6) = 3$
$d(9,7) = 1$	$d(10,7) = 1$	$d(11,7) = 2$	$d(12,7) = 2$

$d(9,8) = 1$	$d(10,8) = 1$	$d(11,8) = 2$	$d(12,8) = 2$
$d(9,9) = 0$	$d(10,9) = 1$	$d(11,9) = 1$	$d(12,9) = 2$
$d(9,10) = 1$	$d(10,10) = 0$	$d(11,10) = 1$	$d(12,10) = 1$
$d(9,11) = 1$	$d(10,11) = 1$	$d(11,11) = 0$	$d(12,11) = 1$
$d(9,12) = 2$	$d(10,12) = 1$	$d(11,12) = 1$	$d(12,12) = 0$

The eccentricity of vertex i is $e(i) = max\{d(i,j):(i,j) \in V(G)\}$

$e(1) = max\{d(1,j): j \in V\}$

$= max\{0,1,1,2,3,2,3,4,3,4,4,5\} = 5$

$e(2) = max\{d(2,j): j \in V\}$

$= max\{1,0,1,12,2,34,4,4,4,5\} = 5$

$e(3) = max\{d(3,j): j \in V\}$

$= max\{1,1,0,1,2,2,2,3,2,3,3,4\} = 4$

$e(4) = max\{d(4,j): j \in V\}$

$= max\{2,1,1,0,1,1,2,3,2,3,3,3\} = 3$

$e(5) = max\{d(5,j): j \in V\}$

$= max\{3,2,2,1,0,1,1,2,2,2,3,3\} = 3$

$e(6) = max\{d(6,j): j \in V\}$

$= max\{2,2,2,1,1,0,1,2,2,2,2,3\} = 3$

$e(7) = max\{d(7,j): j \in V\}$

$= max\{3,3,2,2,1,1,0,1,1,1,2,2\} = 3$

$e(8) = max\{d(8,j): j \in V\}$

$= max\{4,4,3,3,2,2,1,0,1,1,2,2\} = 4$

$e(9) = max\{d(9,j): j \in V\}$

$= max\{3,4,2,2,2,2,1,0,1,1,2,2\} = 4$

$e(10) = max\{d(10,j): j \in V\}$

$= max\{4,4,3,3,2,2,1,1,1,0,1,1\} = 4$

$e(11) = max\{d(11,j): j \in V\}$

$= max\{4,4,3,3,3,2,2,2,1,1,0,1\} = 4$

$e(12) = max\{d(12,j): j \in V\}$

$= max\{5,5,4,3,3,3,2,2,2,1,1,0\} = 5$

Now the diameter(G) = $max\{e(i): i \in V(G)\}$

$= max\{5,5,4,3,3,3,3,4,4,4,4,5\}$ = 5

Therefore, $|NSDS| = 3$, $d = \frac{\ddot{D}}{n-1} = 0.33$ and $diam(G) = 5$

Thus, $diam(G) > |NSDS| > d$

Theorem 2: Let G be a contained connected interval graph corresponding to an interval family $I, i \in D, j = 1$. then $dia(G) \geq |NSDS| \geq \frac{\bar{d}}{n-1}$.

Proof: Let $I = \{1,2, \dots \dots \dots n\}$ be an interval family and G be an interval graph corresponding to I. If i and j are two intervals in I such that $i \in D, j = 1$ and j

Is contained in i and if there is one or more intervals other than i that intersects j, then we will prove that $NSDS$ occurs in G.

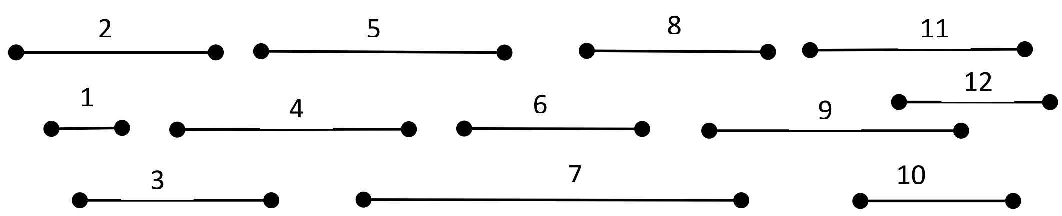
Let $j = 1$ be the interval contained in i, where $i \in D$. Suppose k is an interval , $k \neq i$and k intersects j. Since $i \in D$, $< V - D >$ does not contain i. For that, in $< V - D >$, the vertex j is adjacent to the vertex k. Hence there will not be any disconnection in $< V - D >$. Therefore we get $NSDS$ in G.

Again we will prove that the diameter of G. We have already proved it in theorem 1.

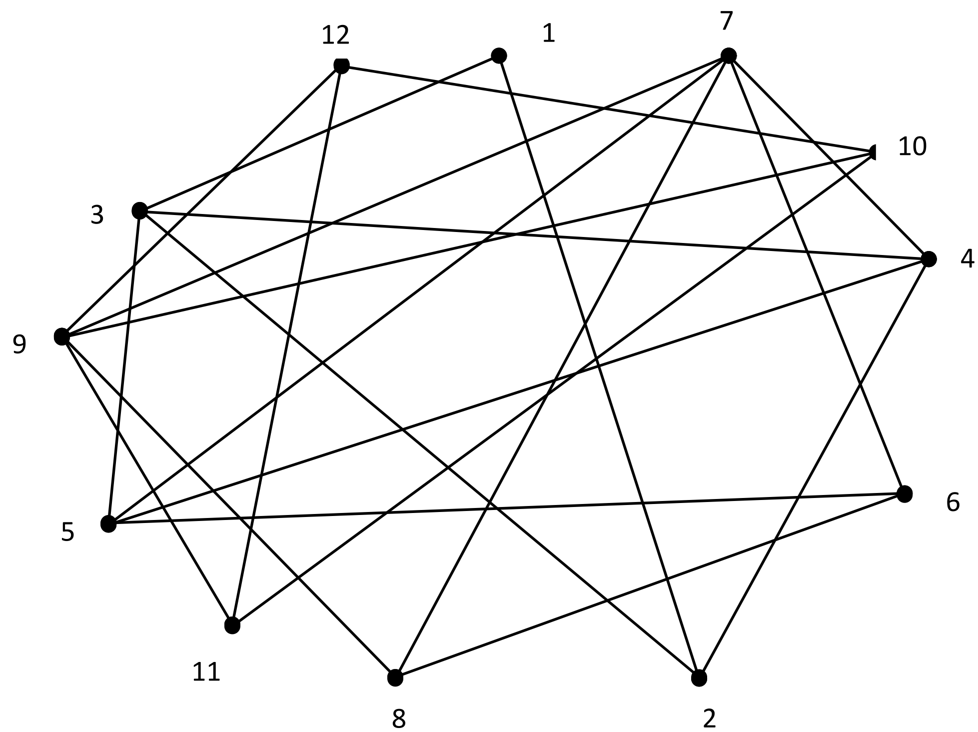
Next we will find the density of G corresponding to an interval family I. This is also proved in theorem1.

Hence $dia(G) \geq |NSDS| \geq \frac{\bar{D}}{n-1}$, at $i \in D, j = 1$.

Illustration 2:



Interval Family(I.F.)



Interval Graph (I.G.).

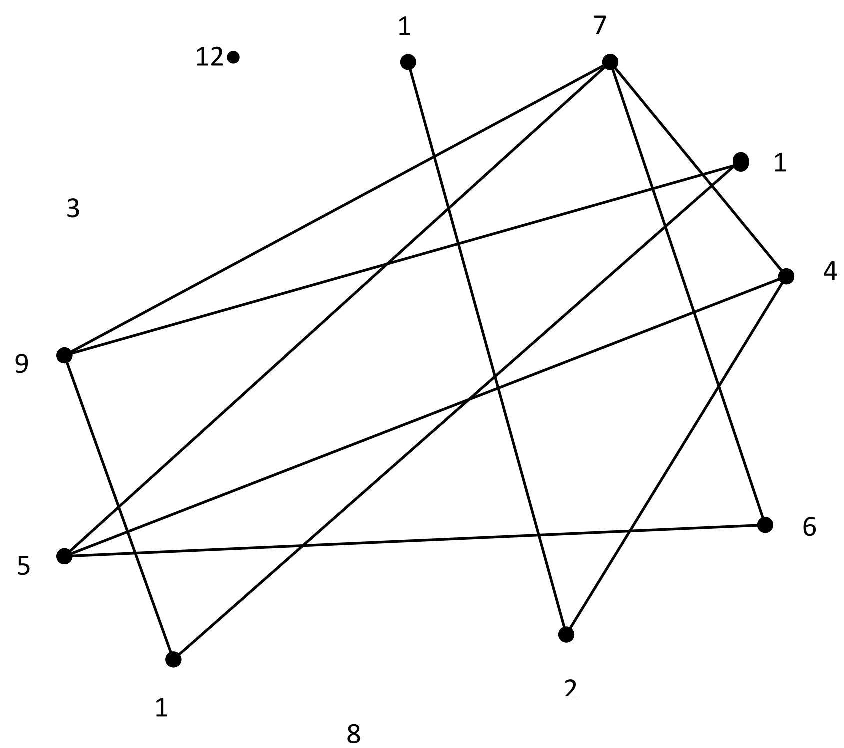
To find dominating set of G:

$I = \{1,2,3,4,5,6,7,8,9,10,11,12\}$,

Dominating set = $\{3,8,12\}$ $\therefore$ the cordinality of DS is $|DS| = 3$

$V - D = \{1,2,4,5,6,7,9,10,11\}$

Induced graph $< V - D >$ is



Interval Graph (I.G.) $-$ G'

From the picture above it is understood that the interval inducede graph G' is connected. The nonsplit dominating set (NSDS) of G' is $\{2,5,9\}$. $\therefore$ $|NSDS| = 3$

To find the average degree of G for density :

deg(1)=1, deg(2)=3, deg(3)=4, deg(4)=4, deg(5)=4, deg(6)=3, deg(7)=5, deg(8)=3, deg(9)=5, deg(10)=3, deg(11)=3, deg(12)=3.

∴ The sum of the degrees of vertices $D = \sum_{i=1}^{12} deg(i) = 40$

Average degree $\overline{D} = \frac{\sum_{i=1}^{12} \deg(i)}{12} = \frac{40}{12} = 3.3$

∴ Density $d = \frac{2E}{n(n-1)} = \frac{\ddot{D}}{n-1} = \frac{40}{12X11} = 0.3$

To find the diameter of $G = max\{e(i) : i \in V(G)\}$

The diameter of vertices $d(i,j)$ are as follows, whwre $i,j \in V(G)$

$d(1,1) = 0$	$d(2,1) = 1$	$d(3,1) = 1$	$d(4,1) = 2$
$d(1,2) = 1$	$d(2,2) = 0$	$d(3,2) = 1$	$d(4,2) = 1$
$d(1,3) = 1$	$d(2,3) = 1$	$d(3,3) = 0$	$d(4,3) = 1$
$d(1,4) = 2$	$d(2,4) = 1$	$d(3,4) = 1$	$d(4,4) = 0$
$d(1,5) = 2$	$d(2,5) = 2$	$d(3,5) = 1$	$d(4,5) = 1$
$d(1,6) = 3$	$d(2,6) = 3$	$d(3,6) = 2$	$d(4,6) = 2$
$d(1,7) = 3$	$d(2,7) = 3$	$d(3,7) = 2$	$d(4,7) = 1$
$d(1,8) = 4$	$d(2,8) = 3$	$d(3,8) = 3$	$d(4,8) = 2$
$d(1,9) = 4$	$d(2,9) = 3$	$d(3,9) = 3$	$d(4,9) = 3$
$d(1,10) = 5$	$d(2,10) = 4$	$d(3,10) = 4$	$d(4,10) = 3$
$d(1,11) = 5$	$d(2,11) = 4$	$d(3,11) = 4$	$d(4,11) = 3$
$d(1,12) = 5$	$d(2,12) = 4$	$d(3,12) = 4$	$d(4,12) = 3$

$d(5,1) = 2$	$d(6,1) = 3$	$d(7,1) = 3$	$d(8,1) = 4$
$d(5,2) = 2$	$d(6,2) = 3$	$d(7,2) = 3$	$d(8,2) = 3$
$d(5,3) = 1$	$d(6,3) = 2$	$d(7,3) = 2$	$d(8,3) = 3$
$d(5,4) = 1$	$d(6,4) = 2$	$d(7,4) = 1$	$d(8,4) = 2$
$d(5,5) = 0$	$d(6,5) = 1$	$d(7,5) = 1$	$d(8,5) = 2$
$d(5,6) = 1$	$d(6,6) = 0$	$d(7,6) = 1$	$d(8,6) = 1$
$d(5,7) = 1$	$d(6,7) = 1$	$d(7,7) = 0$	$d(8,7) = 1$
$d(5,8) = 2$	$d(6,8) = 1$	$d(7,8) = 1$	$d(8,8) = 0$
$d(5,9) = 2$	$d(6,9) = 2$	$d(7,9) = 1$	$d(8,9) = 1$
$d(5,10) = 3$	$d(6,10) = 3$	$d(7,10) = 2$	$d(8,10) = 2$
$d(5,11) = 3$	$d(6,11) = 3$	$d(7,11) = 2$	$d(8,11) = 2$

$d(5,12) = 3$	$d(6,12) = 3$	$d(7,12) = 2$	$d(8,12) = 2$

$d(9,1) = 4$	$d(10,1) = 5$	$d(11,1) = 5$	$d(12,1) = 5$
$d(9,2) = 3$	$d(10,2) = 4$	$d(11,2) = 4$	$d(12,2) = 4$
$d(9,3) = 3$	$d(10,3) = 4$	$d(11,3) = 4$	$d(12,3) = 4$
$d(9,4) = 3$	$d(10,4) = 3$	$d(11,4) = 3$	$d(12,4) = 3$
$d(9,5) = 2$	$d(10,5) = 3$	$d(11,5) = 3$	$d(12,5) = 3$
$d(9,6) = 2$	$d(10,6) = 3$	$d(11,6) = 3$	$d(12,6) = 3$
$d(9,7) = 1$	$d(10,7) = 2$	$d(11,7) = 2$	$d(12,7) = 2$
$d(9,8) = 1$	$d(10,8) = 2$	$d(11,8) = 2$	$d(12,8) = 2$
$d(9,9) = 0$	$d(10,9) = 1$	$d(11,9) = 1$	$d(12,9) = 1$
$d(9,10) = 1$	$d(10,10) = 0$	$d(11,10) = 1$	$d(12,10) = 1$
$d(9,11) = 1$	$d(10,11) = 1$	$d(11,11) = 0$	$d(12,11) = 1$
$d(9,12) = 1$	$d(10,12) = 1$	$d(11,12) = 1$	$d(12,12) = 0$

The eccentricity of vertex i is $e(i) = max\{d(i,j):(i,j) \in V(G)\}$

$e(1) = max\{d(1,j): j \in V\}$

$= max\{0,1,1,2,2,3,3,4,4,5,5,5\} = 5$

$e(2) = max\{d(2,j): j \in V\}$

$= max\{1,0,1,1,2,3,3,3,3,4,4,4\} = 4$

$e(3) = max\{d(3,j): j \in V\}$

$= max\{1,1,0,1,1,2,2,3,3,4,4,4\} = 4$

$e(4) = max\{d(4,j): j \in V\}$

$= max\{2,1,1,0,1,2,1,2,3,3,3,3\} = 3$

$e(5) = max\{d(5,j): j \in V\}$

$= max\{2,2,1,1,0,1,1,2,2,3,3,3\} = 3$

$e(6) = max\{d(6, j): j \in V\}$

$= max\{3,3,2,2,1,0,1,1,2,3,3,3\} = 3$

$e(7) = max\{d(7, j): j \in V\}$

$= max\{3,3,2,1,1,1,0,1,1,2,2,2\} = 3$

$e(8) = max\{d(8, j): j \in V\}$

$= max\{4,3,3,2,2,1,1,0,1,2,2,2\} = 4$

$e(9) = max\{d(9, j): j \in V\}$

$= max\{4,3,3,3,2,2,1,1,0,1,1,1\} = 4$

$e(10) = max\{d(10, j): j \in V\}$

$= max\{5,4,4,3,3,3,2,2,0,1,1,1\} = 5$

$e(11) = max\{d(11, j): j \in V\}$

$= max\{5,4,4,3,3,3,2,2,1,1,0,1\} = 5$

$e(12) = max\{d(12, j): j \in V\}$

$= max\{5,4,4,3,3,3,2,2,1,1,1,0\} = 5$

Now the diameter$(G) = max\{e(i): i \in V(G)\}$

$= max\{5,4,4,3,3,3,3,4,4,5,5,5\} = 5$

Therefore, $|NSDS| = 3$, $d = \frac{\ddot{D}}{n-1} = 0.3$ and $\mathrm{diam}(G) = 5$

Thus, $\mathrm{diam}(G) > |NSDS| > d$

Theorem 3: Let $I = \{1,2,3, \dots \dots, n\}$ be an interval family and G is a connected interval graph. If i, j, k are three consecutive intervals such that $i < j < k$, and $j \in D. i \cap j, j \cap k$ and $k \cap i$. Then $\text{diam}(G) \geq |NSDS| \geq \frac{\bar{D}}{n-1}$.

Proof: We consider G be an interval graph corresponding to an interval family I= $\{1,2,3, \dots \dots, n\}$. If i, j, k are three consecutive intervals such that i$< j < k$, and $j \in D,\ i \cap j, j \cap k$ and $k \cap i$, we have to show that G must be connected as well as G is a non-split dominating set which can be denoted by NSDS.

Let i, j, k be three consecutive intervals satisfying the hypothesis.

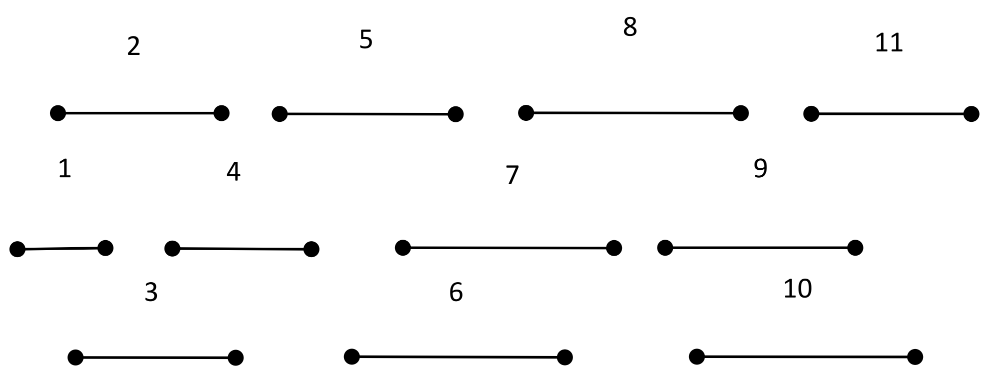
Now i, k intersects implies that i and k are adjacent in $< V - D >$ so that, there will not be any disconnection in $< V - D >$. Therefore G must be a connected graph corresponding to an internal family I.

Again we find the diameter of G corresponding to the consecutive interval family I. Already we have proved the diameter in G in theorem 1.

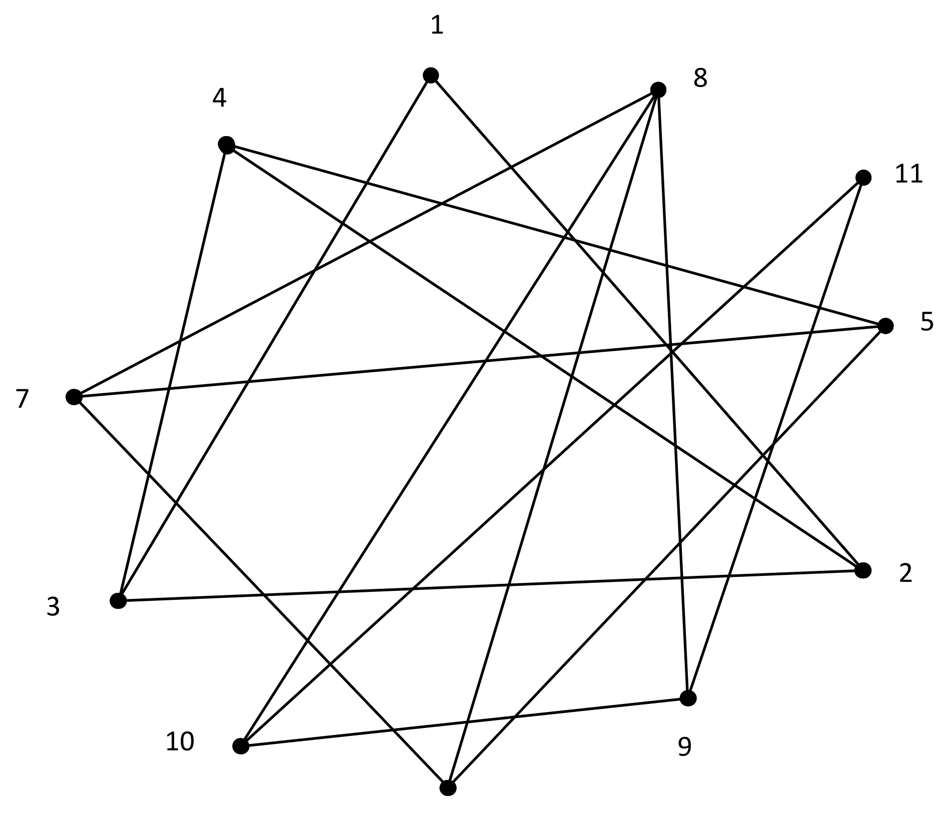
Next we have to prove the density d of G from a consecutive interval family I This proof is already proved in theorem1.

Therefore from the cases said above we get $\text{diam}(G) \geq |NSDS| \geq \frac{\bar{D}}{n-1}$.

Illustration 3:



Interval Family (I.F.)



Interva ıph (I.G.)-G'

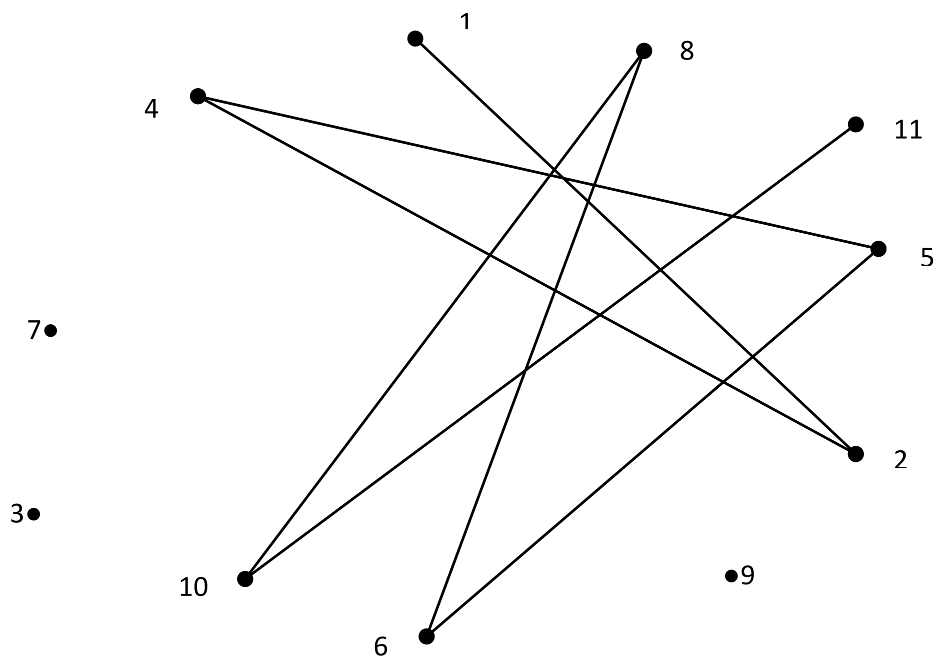
To find dominating set of G:

$I = \{1,2,3,4,5,6,7,8,9,10,11\}$,

Dominating set = $\{3,7,9\}$

$\therefore |DS| = 3$

$$< V - D >= \{1,2,4,5,6,8,10,11\}$$



Interval Graph (I.G.) $-$ G'

The induced subgraph G' is also connected. Hence, it is Non-Split graph. The Non-Split Dominating set (NSDS) is $\{2,6,10\}$ $\therefore$ $|NSDS| = 3$

From the picture above it is understood that the interval graph is connected and G' is non-split dominating set of G.

To find the average degree of G for density:

deg(1)=2, deg(2)=3, deg(3)=3, deg(4)=3, deg(5)=3, deg(6)=3, deg(7)=3, deg(8)=4, deg(9)=3, deg(10)=3, deg(11)=2

$\therefore$ The sum of the degrees of vertices $D = \sum_{i=1}^{11} deg(i)$ = 32.

Average degree $\overline{D} = \frac{\sum_{i=1}^{11} \deg(i)}{11} = \frac{32}{11} = 2.9$

$\therefore$ Density $d = \frac{\ddot{D}}{n-1} = \frac{2E}{n(n-1)} = \frac{2X16}{11X10} = \frac{32}{110} = 0.29$

To find the diameter of $G = max\{e(i) : i \in V(G)\}$

The diameter of vertices $d(i, j)$ are as follows, where $i, j \in V(G)$

$d(1,1) = 0$	$d(2,1) = 1$	$d(3,1) = 1$	$d(4,1) = 2$
$d(1,2) = 1$	$d(2,2) = 0$	$d(3,2) = 1$	$d(4,2) = 1$
$d(1,3) = 1$	$d(2,3) = 1$	$d(3,3) = 0$	$d(4,3) = 1$
$d(1,4) = 2$	$d(2,4) = 1$	$d(3,4) = 1$	$d(4,4) = 0$
$d(1,5) = 3$	$d(2,5) = 2$	$d(3,5) = 2$	$d(4,5) = 1$
$d(1,6) = 3$	$d(2,6) = 3$	$d(3,6) = 2$	$d(4,6) = 2$
$d(1,7) = 4$	$d(2,7) = 3$	$d(3,7) = 3$	$d(4,7) = 2$
$d(1,8) = 5$	$d(2,8) = 4$	$d(3,8) = 4$	$d(4,8) = 3$
$d(1,9) = 6$	$d(2,9) = 5$	$d(3,9) = 5$	$d(4,9) = 4$
$d(1,10) = 6$	$d(2,10) = 5$	$d(3,10) = 5$	$d(4,10) = 4$
$d(1,11) = 7$	$d(2,11) = 6$	$d(3,11) = 6$	$d(4,11) = 5$

$d(5,1) = 3$	$d(6,1) = 3$	$d(7,1) = 4$	$d(8,1) = 5$
$d(5,2) = 2$	$d(6,2) = 3$	$d(7,2) = 3$	$d(8,2) = 4$
$d(5,3) = 2$	$d(6,3) = 2$	$d(7,3) = 3$	$d(8,3) = 4$
$d(5,4) = 1$	$d(6,4) = 2$	$d(7,4) = 2$	$d(8,4) = 3$
$d(5,5) = 0$	$d(6,5) = 1$	$d(7,5) = 1$	$d(8,5) = 2$
$d(5,6) = 1$	$d(6,6) = 0$	$d(7,6) = 1$	$d(8,6) = 1$
$d(5,7) = 1$	$d(6,7) = 1$	$d(7,7) = 0$	$d(8,7) = 1$
$d(5,8) = 2$	$d(6,8) = 1$	$d(7,8) = 1$	$d(8,8) = 0$
$d(5,9) = 3$	$d(6,9) = 2$	$d(7,9) = 2$	$d(8,9) = 1$
$d(5,10) = 3$	$d(6,10) = 2$	$d(7,10) = 2$	$d(8,10) = 1$
$d(5,11) = 4$	$d(6,11) = 3$	$d(7,11) = 3$	$d(8,11) = 2$

$d(9,1) = 6$	$d(10,1) = 6$	$d(11,1) = 7$
$d(9,2) = 5$	$d(10,2) = 5$	$d(11,2) = 6$
$d(9,3) = 5$	$d(10,3) = 5$	$d(11,3) = 6$
$d(9,4) = 4$	$d(10,4) = 4$	$d(11,4) = 5$
$d(9,5) = 3$	$d(10,5) = 3$	$d(11,5) = 4$
$d(9,6) = 2$	$d(10,6) = 2$	$d(11,6) = 3$
$d(9,7) = 2$	$d(10,7) = 2$	$d(11,7) = 3$

$d(9,8) = 1$	$d(10,8) = 1$	$d(11,8) = 2$
$d(9,9) = 0$	$d(10,9) = 1$	$d(11,9) = 1$
$d(9,10) = 1$	$d(10,10) = 0$	$d(11,10) = 1$
$d(9,11) = 1$	$d(10,11) = 1$	$d(11,11) = 0$

The eccentricity of vertex i is $e(i) = max\{d(i,j):(i,j) \in V(G)\}$

$e(1) = max\{d(1,j): j \in V\}$

$= max\{0,1,1,2,3,3,4,5,6,6,7\} = 7$

$e(2) = max\{d(2,j): j \in V\}$

$= max\{1,0,1,1,2,3,3,4,5,5,6\} = 6$

$e(3) = max\{d(3,j): j \in V\}$

$= max\{1,1,0,1,2,2,3,4,5,5,6\} = 6$

$e(4) = max\{d(4,j): j \in V\}$

$= max\{2,1,1,0,1,2,2,3,4,4,5\} = 5$

$e(5) = max\{d(5,j): j \in V\}$

$= max\{3,2,21,0,1,1,2,3,3,4\} = 4$

$e(6) = max\{d(6,j): j \in V\}$

$= max\{3,3,2,2,1,0,1,1,2,2,3\} = 3$

$e(7) = max\{d(7,j): j \in V\}$

$= max\{4,3,3,2,1,1,0,1,2,2,3\} = 4$

$e(8) = max\{d(8,j): j \in V\}$

$= max\{5,4,4,3,2,1,1,0,1,1,2\} = 5$

$e(9) = max\{d(9,j): j \in V\}$

$= max\{6,5,5,4,3,2,2,1,0,1,1\} = 6$

$e(10) = max\{d(10,j): j \in V\}$

$= max\{6,5,5,4,3,2,2,1,1,0,1\} = 6$

$e(11) = max\{d(11,j): j \in V\}$

$= max\{7,6,6,5,4,3,3,2,1,1,0\} = 7$

$\therefore$ The diameter$(G) = max\{e(i): i \in V(G)\}$

$= max\{7,6,6,5,4,3,4,5,6,6,7\} = 7$

Therefore, $|NSDS| = 3$, $d = \frac{\ddot{D}}{n-1} = 0.29$ and $diam(G) = 7$

Thus, $diam(G) > |NSDS| > d$

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