

A NEW STUDY ON THE LINEAR ALGEBRA

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ABSTRACT

Linear algebra is a main important part of mathematics. The branch of mathematics dealing with the study of vectors, vector spaces, Linear maps and linear equation structures is linear algebra. In modern mathematics, vector spaces are a central theme; Linear algebra is also commonly used in both abstract algebra and functional analysis. In analytic geometry, linear algebra also has concrete representation and in operator theory it is generalized. In the natural science and social science, it has broad application since nonlinear models can often be approximated by Linear ones

KEYWORDS: *Linear algebra, Linear Transformation, vector spaces, Linear combination*

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INTRODUCTION:

It is a principal branch of mathematics that is related to mathematical structure closed under the operations of addition and scalar multiplication and that includes the theory of system of linear equation, matrices, determinants, vectors spaces and linear transformation. Linear algebra ,is a mathematical discipline that deals with vectors and matrices and more generally ,with vector space and linear transformation .It is a key concept for almost all areas of mathematics .Linear algebra is considered a basic concept in the modern presentation of geometry .It is mostly used in physics and Engineering as it helps to define the basic objects such as planes ,lines and rotations of the object .It allows us to model many natural phenomena, and also it has a computing efficiency.

Linear algebra had it beginning in the study of vectors in Cartesian 2-space and 3-space. A vector, here, is a directed line segment characterized by both its magnitude, represented by length, and its direction. vectors can be used to represent physical entities such as forces, and they can be added to each other and multiplied with scalars, thus

forming the first example of a real vector space.

Modern linear algebra has been extended to consider spaces of arbitrary or infinite dimension. A vectors space of dimension n is called n -space. Most of the useful results from 2-and3-space can be extended to these higher dimensional spaces. Although people cannot easily visualize vectors in n - space. since vectors, as n – tuples, are ordered lists of n components, it is possible to summarize and manipulate data efficiently in this framework.

for example, in economics, one can create and use, say 8-vectors or 8-tuples to represent the Gross National product of 8 countries One can decide to display the GNP of 8 countries for a particular year, where the countries' order is specified, for example, (United States, United Kingdom, France, Germany, Spain, India, Japan, Australia), by using a vector $(V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8)$ where each country 's GNP is in its respective position

Linear algebra is one of the most known mathematical disciplines because of its rich theoretical foundations and its many useful application to science and engineering, solving system of linear equations and computing determinants are two examples of fundamental problems in linear algebra that have been studied for a long time ago. Leibnitz found the formula for determinants in 1693 and in 1750 crammer presented a method for solving system of linear equations, which is today known as crammer rule

LINEAR TRANSFORMATION:

Nicosia, 2016 conducted the study on function of linear algebra such as vectors as both input and output. they observed that in linear algebra vectors are objects that can be added or scalar multiplied. A linear transformation T from U to V , is a capacity component of the vector space U (called the domain) to the vector space V (called the co domain) and which has two extra properties

- [1] $T(u_1+u_2) = T(u_1) + T(u_2)$ for all u_1, u_2 belong to U
- [2] $T(a U) = a. T(U)$ for all u belong to U and all a belong to C

Dummit,2016 observed that if V and W are vector spaces, we say a map T from V to W is a linear transformation if, for any vector v, V_1, V_2 and any scalar a , the following two properties hold:

- 1) The map respect addition of vectors: $T(v_1+v_2) = T(v_1) + T(v_2)$
- 2) The map respect scalar multiplication $T(a v) = a. T(v)$

It is important to remember that the addition on the left side occurs within V in the statement $T(v_1+v_2) = T(v_1) + T(v_2)$ while the addition on the right side take place within W . similarly, the scalar multiplication on the left-hand side is in V in the Statement $T(a v) = a T(v)$, while the scalar multiplication on the right -hand side is in W if V is the vector space of all differentiable function and W is the vector space of all functions, decide if a linear transformation from V to W is the derivative map that sends a function to its derivative

(T1): we have $D(f_1+f_2) = D(f_1) + D(f_2)$ (T2): Also, $D(a. f) = a. D(f)$

Since both parts of definition are satisfied, the derivative is a linear transformation

PROPERTIES OF LINEAR TRANSFORMATION:

Gilbert, 2009 gives four properties of linear transformation. Let V and W be two vector spaces. Suppose $T: V \rightarrow W$ is linear transformation. then

$$1) T(0) = 0$$

$$T(-v) = -T(v) \text{ for all } v \text{ belong } V$$

$$T(u-v) = T(u) - T(v) \text{ for all } u, v \text{ belongs } V$$

$$4) \text{ If } v = c_1v_1 + c_2v_2 + \dots + c_nv_n \text{ then } T(v) = T(c_1v_1 + c_2v_2 + \dots + c_nv_n) = c_1T(v_1) + c_2T(v_2) + \dots + c_nT(v_n)$$

VECTOR SPACE:

A real vector space is a set V of elements on which we have two operations $(+)$ and (\cdot) defined with the following properties

If u and v are any elements in V , then $u+v$ is in V , (we say that V is closed under the operation $+$)

$$(1) u+v = v+u \text{ for all } u, v \text{ in } V$$

$$(2) u+(v+w) = (u+v)+w \text{ for all } u, v, w \text{ in } V$$

$$(3) \text{ There exists an element } -u \text{ in } V \text{ such that } u+(-u) = -u+u = 0$$

$$(4) \text{ there exists an element } 0 \text{ in } V \text{ such that } u+0 = 0+u = u \text{ for all } u \text{ belong } V, \text{ this element '0' is called zero vector}$$

(b) If u is any element in V and c is any real number, then $c \cdot u$ is in V (V is closed under the operations multiplication)

$$(5) a(u+v) = au + av \text{ for any } u, v \text{ in } V \text{ and any real number } a$$

$$(6) (a+b)u = au + bu \text{ for any } u \text{ in } V \text{ and any real number } a \text{ and } b$$

$$(7) (ab)u = a(bu) \text{ for any } u \text{ in } V \text{ and any real number } a \text{ and } b$$

$$(8) 1 \cdot u = u \text{ for any } u \text{ in } V \text{ and '1' is the multiplicative identity of } F$$

The elements of V are called vectors: the elements of the set of real number R are called scalars. the operations $(+)$ is called vector addition: the operation (\cdot) is called scalar multiplication

Theorem: Let V be a vector space with operations $(+)$ and (\cdot) and W be non-empty subset of V . then W is a subspace of V if and only if the following condition hold (a) if u and v are any vector in w , then $u+v$ is in w

$$(b) \text{ if } c \text{ is any real number and } u \text{ is any vector in } W, \text{ then } c \cdot u \text{ in } w$$

LINEAR EQUATION:

A linear equations is an algebraic equation in which each term is either a constant or the product of a constant and (the first power of) a single variable. Linear equations can have one or more variable. Linear equations occur abundantly in most subareas of mathematics and especially in applied mathematics. while they are particularly useful since many nonlinear equation may be reduced to linear equation by assuming that quantities of interest vary

to only a small extent for some "background" state. Linear equation does not include exponents. A particular case of matrix multiplication is rightly linked to linear equation: if x designates a column vector ($n \times 1$ - matrix) of n variable x_1, x_2, \dots, x_n and A is an m by n matrix then the matrix equation

$$AX=b$$

Where b is some $m \times 1$ - column vector, is equivalent to the system of linear equations $A_{11} X_1 + A_{12} X_2 + \dots + A_{1n} X_n = b_1$
 $A_{m1} X_1 + A_{m2} X_2 + \dots + A_{mn} X_n = b_m$

This way, matrices can be used to compactly write and deal with multiple linear equations (system of linear equations)

LINEAR COMBINATION:

For the most part, mathematics, you say that a linear combination of things is an entirety of products of those things (Poole, 2010) Along lines, for instance, one linear combination of the functions $f(x)$, $g(x)$ and $h(x)$ is $2f(x) + 3g(x) - 4h(x)$

Definition: Let $V(F)$ be a vector space. A vector v belong to V to be linear combination of vectors $v_1, v_2, v_3, \dots, v_k$ belong to vector space V if v expressed $v = c_1 v_1 + c_2 v_2 + \dots + c_k v_k$, where C_i 's are scalars (Larry 1998) exhibit

CONCLUSION:

Linear algebra is the branch of mathematics dealing with the study of vectors. In this we are presenting a new study on the linear algebra. A linear equation is an algebraic equation in which each term is either constant or the product of a constant and (the first power of) a single variable

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